Let $C$ be given by the vector function
\[ \vec{r}(t) = g(t) \mathbf{i} + h(t) \mathbf{j} + k(t) \mathbf{k} \]
for $t = a$ to $t = b$. Define function $w = f(x, y, z)$ on curve $C$ by $w = f(g(t), h(t), k(t))$.

Divide curve $C$ into $N$ parts $C_1, C_2, C_3, \ldots, C_n$ of arc length $\Delta S_1, \Delta S_2, \Delta S_3, \ldots, \Delta S_n$. Pick sampling points $P_i = (x_i, y_i, z_i)$ in $C_i$ for $i = 1, 2, 3, \ldots, N$. We can now define the following new integral.

Def: The line integral of $f$ over curve $C$ from $t = a$ to $t = b$ is
\[
\int_C f(x, y, z) \, ds = \lim_{\text{mesh} \to 0} \sum_{i=1}^{n} f(x_i, y_i, z_i) \cdot ds_i
\]

Method of Evaluation:
If \( C \) is determined by
\[
\vec{r}(t) = g(t) \vec{i} + h(t) \vec{j} + k(t) \vec{k}
\]
for \( a \leq t \leq b \), then
\[
\int_C f(x, y, z) \, ds = \int_{a}^{b} f(g(t), h(t), k(t)) \cdot \frac{d\vec{r}}{dt} \, dt,
\]
where
\[
\frac{d\vec{r}}{dt} = |\vec{r}'(t)| = \sqrt{(g'(t))^2 + (h'(t))^2 + (k'(t))^2}.
\]

Applications of Line Integrals
Assume that \( \delta(x, y, z) \) is the density (mass/length units) at point \( P = (x, y, z) \) on curve \( C \) determined by
\[
\vec{r}(t) = g(t) \vec{i} + h(t) \vec{j} + k(t) \vec{k}
\]
for \( t = a \) to \( t = b \).

1.) Length = \( \int_C ds \)

2.) Mass = \( \int_C \delta(x) \, ds \)
3.) Moments about planes:

a.) \( M_{x=a} = \int_C (x-a) \delta(p) \, ds \)

b.) \( M_{y=a} = \int_C (y-a) \delta(p) \, ds \)

c.) \( M_{z=a} = \int_C (z-a) \delta(p) \, ds \)

4.) Center of Mass \((\bar{x}, \bar{y}, \bar{z})\):

\[
\bar{x} = \frac{\int_C x \delta(p) \, ds}{\int_C \delta(p) \, ds}, \quad \bar{y} = \frac{\int_C y \delta(p) \, ds}{\int_C \delta(p) \, ds}, \quad \bar{z} = \frac{\int_C z \delta(p) \, ds}{\int_C \delta(p) \, ds}
\]

5.) Centroid \((\bar{x}, \bar{y}, \bar{z})\):

\[
\bar{x} = \frac{\int_C x \, ds}{\int_C 1 \, ds}, \quad \bar{y} = \frac{\int_C y \, ds}{\int_C 1 \, ds}, \quad \bar{z} = \frac{\int_C z \, ds}{\int_C 1 \, ds}
\]

6.) Moment of Inertia:

\( M \text{. of I.} = \int_C (\text{distance})^2 \delta(p) \, ds \)

where distance refers to the distance from point \( P = (x, y, z) \) on curve \( C \) to either a point or axis of rotation.