Math 21D (Summer Session I 2020)
Kouba
Quiz 3

Printing and signing your name below is a verification that no other person assisted you in the completion of this Quiz.

PRINT your name KEY SIGN your name

Show clear, organized supporting work for your answers. Correct answers without supporting work may not receive full credit. Use of unapproved shortcuts may not receive full credit. There are 17 pages, including blank pages for work space. You must submit 17 pages to Gradescope.

1.) (12 pts.) Consider the vector field

$$\vec{F}(x,y,z) = \underbrace{(3x^2z+3+yz)}_{M}\vec{i} - \underbrace{(z^2-xz)}_{N}\vec{j} + \underbrace{(x^3-2yz+2+xy)}_{P}\vec{k}$$

Find a scalar function f, which has a gradient vector equal to $\vec{F}$, or determine that this is impossible.

$M_Z = 3X^2 + Y = P_X$, $M_Y = Z = N_X$, $N_Z = X - 2Z = P_Y$, so $\vec{F}$ is a gradient vector field. assume that

$f_X = 3X^2Z + 3 + YZ \xrightarrow{\int_X}$

$f = X^3Z + 3X + XYZ + k(Y,Z) \xrightarrow{D_Y}$

$f_Y = 0 + 0 + XZ + k_Y(Y,Z)$

$= XZ + k_Y(Y,Z)$

$= XZ - Z^2 \longrightarrow k_Y(Y,Z) = -Z^2$

$\xrightarrow{\int_Y} k(Y,Z) = -YZ^2 + g(Z) \longrightarrow$

$$f = x^3 z + 3x + xyz - yz^2 + g(z) \xrightarrow{D_z}$$

$$f_z = x^3 + 0 + xy - 2yz + g'(z)$$

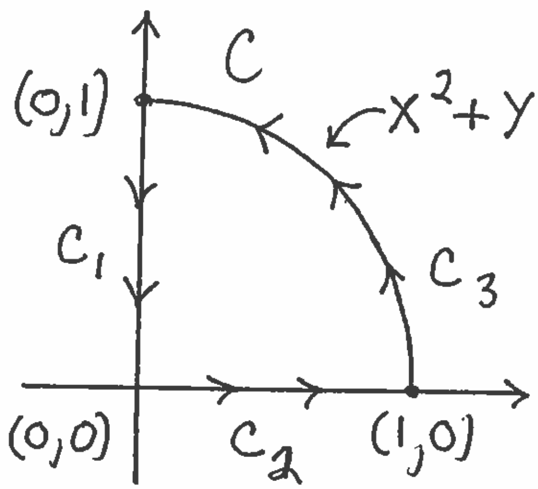
$$= x^3 - 2yz + 2 + xy \longrightarrow$$

$$g'(z) = 2 \xrightarrow{\int} g(z) = 2z + \cancel{C}^{\,0} \longrightarrow$$

$$\boxed{f(x,y,z) = x^3 z + 3x + xyz - yz^2 + 2z} \, .$$

2.) (12 pts.) Consider the velocity vector field $\vec{F}(x,y) = (y)\vec{i} - (x)\vec{j}$ (units: gm/(cm)(sec)). Find the Circulation (Flow) of $\vec{F}$ along path C given in the diagram below.

$$\text{Flow} = \oint_C \vec{F}\bullet\vec{T}\,ds$$



$$= \int_{C_1} \vec{F}\bullet\vec{T}\,ds + \int_{C_2} \vec{F}\bullet\vec{T}\,ds + \int_{C_3} \vec{F}\bullet\vec{T}\,ds$$

$$= A + B + D \quad ;$$

I.) C_1 : $\begin{cases} x = 0 \\ y = 1 - t \end{cases}$ for $0 \le t \le 1$, so

$$A = \int_{C_1} \vec{F}\bullet\vec{T}\,ds = \int_{C_1} \left(M\frac{dx}{dt} + N\frac{dy}{dt}\right)dt$$

$$= \int_{C_1} [(y)(0) + (-\overset{0}{x})(-1)]\,dt = \int_0^1 0\,dt = 0 \;;$$

II.) $C_2: \begin{cases} x = t \\ y = 0 \end{cases}$ for $0 \le t \le 1$, so

$$B = \int_{C_2} \vec{F} \cdot \vec{T}\, ds = \int_{C_2} \left(M \frac{dx}{dt} + N \frac{dY}{dt} \right) dt$$

$$= \int_{C_2} [(\cancel{y}^{\,0})(1) + (-x)(0)]\, dt$$

$$= \int_0^1 0\, dt = 0\,;$$

III.) $C_3: \begin{cases} x = \cos t \\ y = \sin t \end{cases}$ for $0 \le t \le \frac{\pi}{2}$, so

$$D = \int_{C_3} \vec{F} \cdot \vec{T}\, ds = \int_{C_3} \left[M \frac{dx}{dt} + N \frac{dY}{dt} \right] dt$$

$$= \int_{C_3} [(y)(-\sin t) + (-x)(\cos t)]\, dt$$

$$= \int_0^{\pi/2} [(\sin t)(-\sin t) + (-\cos t)(\cos t)]\, dt$$

$$= \int_0^{\frac{\pi}{2}} [-(\sin^2 t + \cos^2 t)]\, dt = \int_0^{\frac{\pi}{2}} -1\, dt$$

$$= -t \Big|_0^{\frac{\pi}{2}} = -\frac{\pi}{2},$$ so Total Flow is

$$A + B + D = \boxed{-\frac{\pi}{2} \text{ gm./sec.}}$$

or Green's Theorem 2:

$$\oint_C \vec{F}\cdot\vec{T}\,ds = \iint_R (N_x - M_y)\,dA$$

$$= \iint_R ((-1)-(1))\,dA$$

$$= -2\iint_R 1\,dA$$

$$= -2\,(\text{Area of } R)$$

$$= -2\left(\tfrac{1}{4}\pi(1)^2\right)$$

$$= -\frac{\pi}{2} \quad \text{gm./sec.}$$

3.) (12 pts.) Find the Flux of the velocity vector field $\vec{F}(x,y) = (y^2)\vec{i} + (y)\vec{j}$ along path C given by $x^2 + y^2 = 4$.

$$C : \begin{cases} x = 2\cos t \\ y = 2\sin t \end{cases} \text{ for } 0 \le t \le 2\pi \text{, then}$$

$$\text{Flux} = \int_C \vec{F}\cdot\vec{n}\,ds = \int_C \left(M\frac{dy}{dt} - N\frac{dx}{dt}\right)dt$$

$$= \int_C \left[(y^2)(2\cos t) - (y)(-2\sin t)\right]dt$$

$$= \int_0^{2\pi}\left[(4\sin^2 t)(2\cos t) - (2\sin t)(-2\sin t)\right]dt$$

$$= \int_0^{2\pi}\left[8\sin^2 t\cos t + 4\sin^2 t\right]dt$$

$$= \int_0^{2\pi}\left[8\sin^2 t\cos t + 4\cdot\tfrac{1}{2}(1-\cos 2t)\right]dt$$

$$= \left(\frac{8}{3}\sin^3 t + 2\left(t - \frac{1}{2}\sin 2t\right)\right)\Big|_0^{2\pi}$$

$$= \left(\frac{8}{3}\cancel{\sin^3 2\pi}^{\,0} + 2(2\pi) - \cancel{\sin 4\pi}^{\,0}\right) - (0)$$

$$= \boxed{4\pi}$$

OR use Green's Theorem 1 :

$$\oint_C \vec{F} \cdot \vec{n} \, ds = \iint_R (M_x + N_y) \, dA$$

$$= \iint_R (0+1) \, dA$$

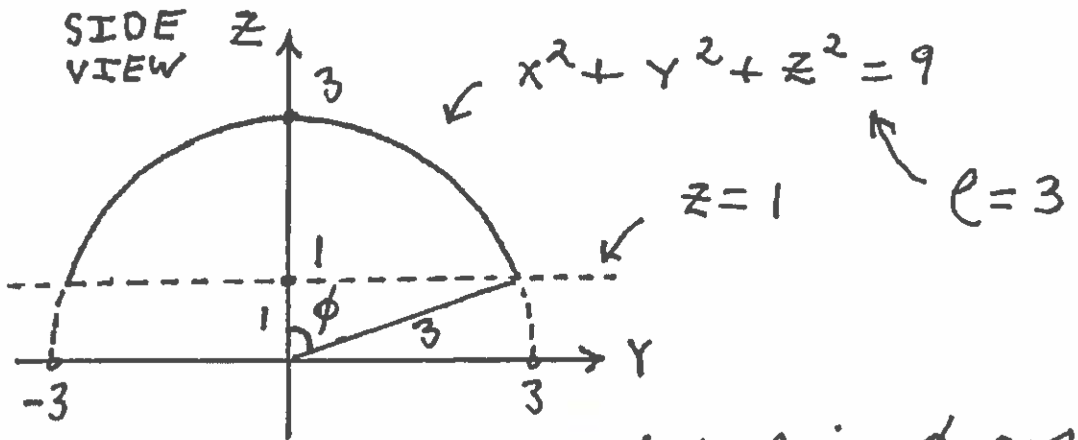
$$= \iint_R 1 \, dA$$

$$= \text{Area of } R$$

$$= \pi (2)^2$$

$$= \boxed{4\pi}$$

4.) (12 pts.) Let Surface S be that portion of the sphere $x^2 + y^2 + z^2 = 9$, which is above the plane $z = 1$. Parametrize this surface and write your final answer in vector function notation.



$$\underline{\text{RECALL}}: \begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases}$$

$$\mathcal{S} : \begin{cases} x = 3 \sin\phi \cos\theta \\ y = 3 \sin\phi \sin\theta \\ z = 3 \cos\phi \end{cases}$$

for $0 \le \theta \le 2\pi$, $0 \le \phi \le \arccos\left(\frac{1}{3}\right)$

$$\boxed{\vec{r}(\theta, \phi) = (3 \sin\phi \cos\theta)\vec{i} + (3 \sin\phi \sin\theta)\vec{j} + (3 \cos\phi)\vec{k}}$$

OR $\left.\begin{aligned} x^2+y^2+z^2 &= 9 \\ z &= 1 \end{aligned}\right\} \longrightarrow$

$$x^2+y^2 = 8 = \left(2\sqrt{2}\right)^2$$

$$S: \begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = \sqrt{9-r^2} \end{cases}$$

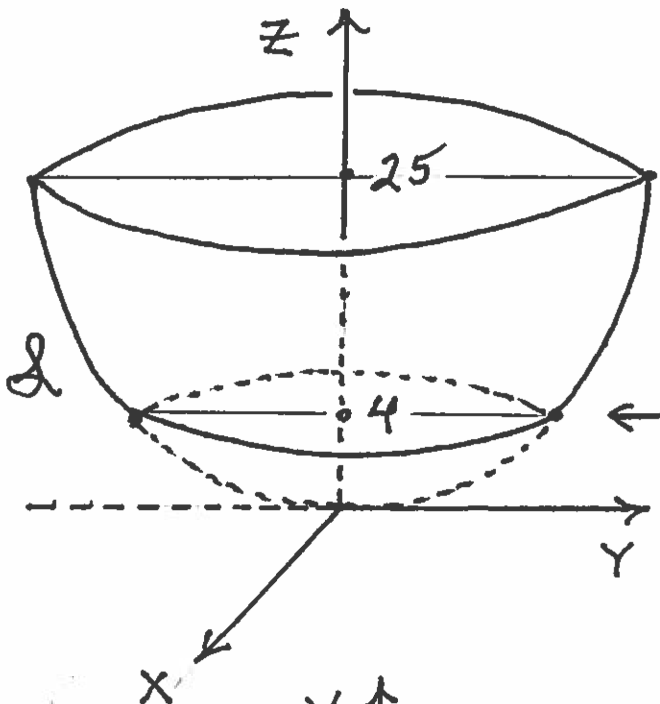
and $0 \le \theta \le 2\pi$, $0 \le r \le 2\sqrt{2}$

$$z^2 = 9-(x^2+y^2)$$

$$\longrightarrow \quad z = \sqrt{9-r^2} \quad \longrightarrow$$

$$\boxed{\vec{r}(r,\theta) = (r\cos\theta)\vec{i} + (r\sin\theta)\vec{j} + \left(\sqrt{9-r^2}\right)\vec{k}}$$

5.) (12 pts.) Let Surface S be that portion of the paraboloid $z = x^2 + y^2$, which lies between the planes $z = 4$ and $z = 25$. Find the Area of S.



$\leftarrow Z = 25$

$\leftarrow Z = X^2 + Y^2 \longrightarrow \underbrace{X^2 + Y^2 - Z}_{f(X,Y,Z)} = 0$

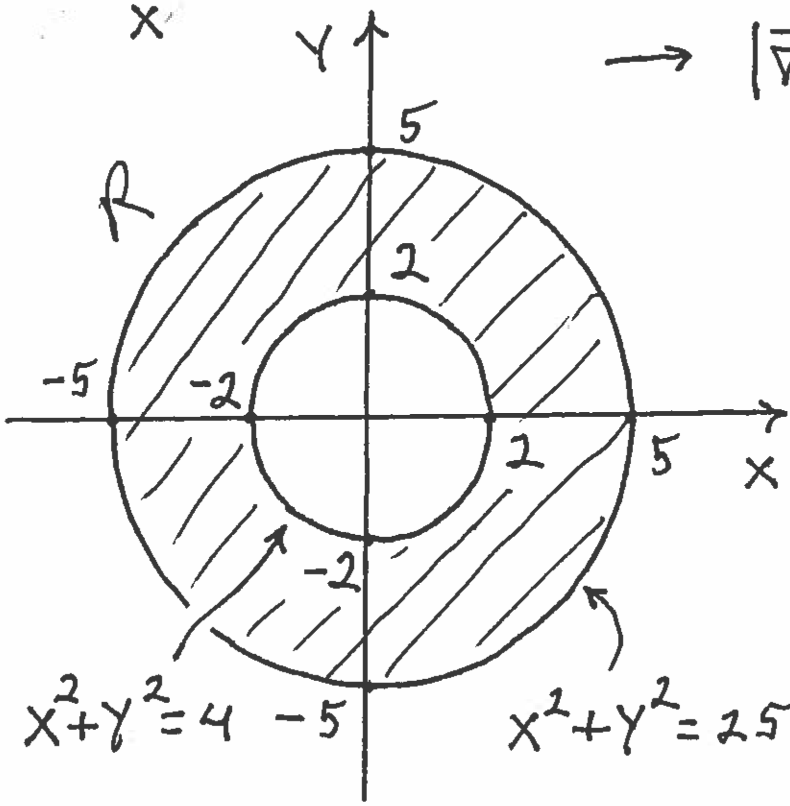
$\leftarrow Z = 4$

$$\vec{\nabla} f = (2X)\vec{i} + (2Y)\vec{j} + (-1)\vec{k}$$

$$\longrightarrow \left|\vec{\nabla} f\right| = \sqrt{(2X)^2 + (2Y)^2 + (-1)^2}$$

$$= \sqrt{4X^2 + 4Y^2 + 1}$$

$$= \sqrt{4(X^2+Y^2)+1},$$



then

$$\text{area of } \mathcal{S} = \iint_{\mathcal{S}} 1 \, dS$$

$$= \iint_R 1 \cdot \frac{|\vec{\nabla} f|}{|f_z|} \, dA$$

$$= \iint_R \frac{\sqrt{4(x^2+y^2)+1}}{|-1|} \, dA$$

$$= \int_0^{2\pi} \int_2^5 \sqrt{4r^2+1} \cdot r \, dr \, d\theta \qquad R: \begin{cases} 0 \le \theta \le 2\pi \\ 2 \le r \le 5 \end{cases}$$

$$= \int_0^{2\pi} \frac{2}{3} \cdot \frac{1}{8} (4r^2+1)^{3/2} \Big|_{r=2}^{r=5} \, d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{12}(101)^{3/2} - \frac{1}{12}(17)^{3/2} \right) d\theta$$

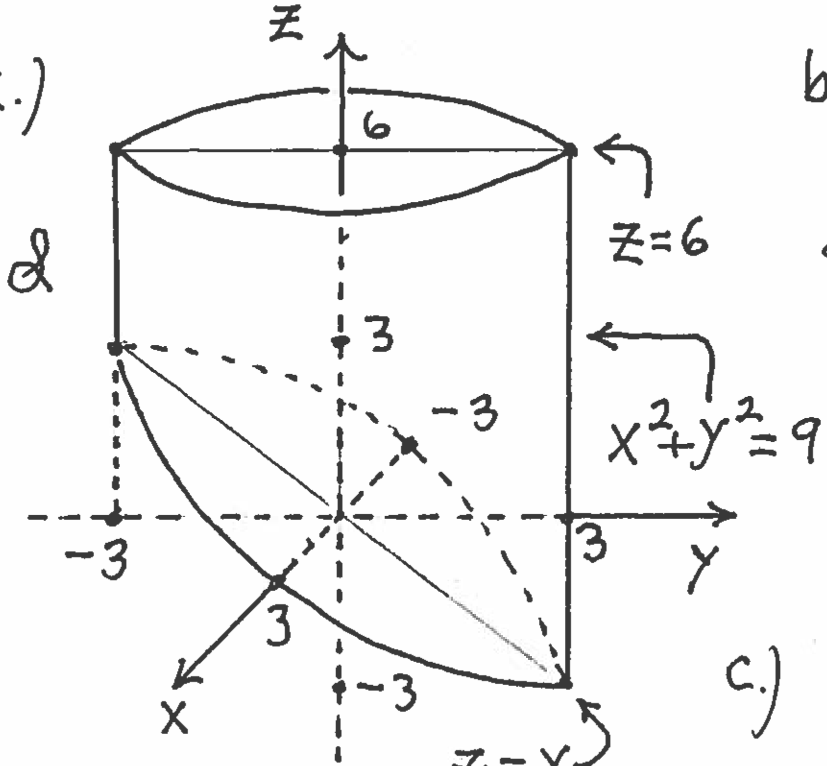
$$= \left(\frac{1}{12}(101)^{3/2} - \frac{1}{12}(17)^{3/2} \right) \cdot \theta \Big|_0^{2\pi}$$

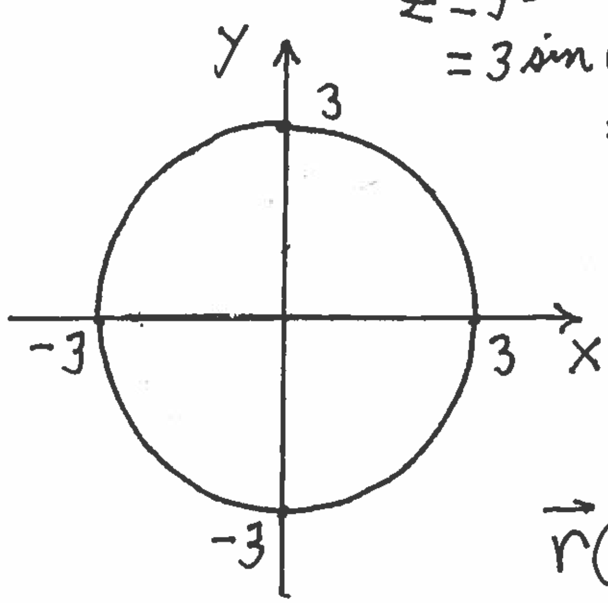
$$= \frac{\pi}{6} \left((101)^{3/2} - (17)^{3/2} \right)$$

6.) (16 pts.) Let Surface S be that portion of the cylinder $x^2 + y^2 = 9$, which lies between the planes $z = y$ and $z = 6$.

a.) Sketch the Surface S.
b.) Parametrize the Surface S.
c.) Evaluate the following Surface Integral: $\iint_{S} (y - z)\, dS$

a.)





b.)

$$S: \begin{cases} x = 3\cos\theta \\ y = 3\sin\theta \\ z = t \end{cases}$$

for $0 \le \theta \le 2\pi$,
$3\sin\theta \le t \le 6$

c.) $\iint_{S} (y - z)\, dS$

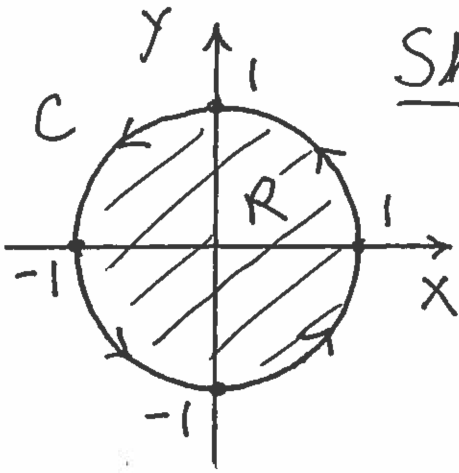
$$= \iint_R (y-z) \left| \vec{r}_\theta \times \vec{r}_t \right| dA,$$

where

$$\vec{r}(\theta,t) = (3\cos\theta)\vec{i} + (3\sin\theta)\vec{j} + (t)\vec{k}$$

$$\vec{r}_\theta = (-3\sin\theta)\vec{i} + (3\cos\theta)\vec{j} + (0)\vec{k},$$
$$\vec{r}_t = (0)\vec{i} + (0)\vec{j} + (1)\vec{k}, \text{ then}$$
$$\vec{r}_\theta \times \vec{r}_t = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3\sin\theta & 3\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$
$$= (3\cos\theta - 0)\vec{i} - (-3\sin\theta - 0)\vec{j} + (0-0)\vec{k}$$
$$= (3\cos\theta)\vec{i} + (3\sin\theta)\vec{j} + (0)\vec{k}; \text{ then}$$
$$|\vec{r}_\theta \times \vec{r}_t| = \sqrt{(3\cos\theta)^2 + (3\sin\theta)^2}$$
$$= \sqrt{9(\cos^2\theta + \sin^2\theta)} = \sqrt{9(1)} = 3 \text{ ; so}$$
$$\iint_R (y-z)\,|\vec{r}_\theta \times \vec{r}_t|\,dA = \int_0^{2\pi}\int_{3\sin\theta}^{6} (3\sin\theta - t)\,dt\,d\theta$$
$$= \int_0^{2\pi} \left((3\sin\theta)t - \tfrac{1}{2}t^2\right)\Big|_{t=3\sin\theta}^{t=6} d\theta$$
$$= \int_0^{2\pi} \left[\left((3\sin\theta)(6) - \tfrac{1}{2}(6)^2\right) - \left((3\sin\theta)^2 - \tfrac{1}{2}(3\sin\theta)^2\right)\right] d\theta$$
$$= \int_0^{2\pi} \left[18\sin\theta - 18 - \tfrac{9}{2}\sin^2\theta\right] d\theta$$
$$= (-18\cos\theta - 18\theta)\Big|_0^{2\pi} - \frac{9}{2}\int_0^{2\pi} \frac{1}{2}(1-\cos 2\theta)\,d\theta$$
$$= (\cancel{-18(1)} - 18(2\pi)) - (\cancel{-18(1)} - 0) - \frac{9}{4}\left(\theta - \tfrac{1}{2}\sin 2\theta\right)\Big|_0^{2\pi}$$
$$= -36\pi - \frac{9}{4}\left[(2\pi - 0) - (0-0)\right]$$
$$= -36\pi - \frac{9}{2}\pi = -\frac{81}{2}\pi .$$

7.) (12 pts.) Verify Green's Theroem 2 for the Vector Field $\vec{F}(x, y) = (xy)\vec{i} + (y^2)\vec{j}$, where the closed curve C is the circle $x^2 + y^2 = 1$.



Show: $\oint_C \vec{F} \cdot \vec{T}\, ds = \iint_R (N_x - M_y)\, dA.$

$$\oint_C \vec{F} \cdot \vec{T}\, ds = \oint_C M\,dx + N\,dy$$

$$= \oint_C [(xy)(-\sin t) + (y^2)(\cos t)]\, dt$$

$C: \begin{cases} x = \cos t \\ y \sin t \end{cases}$ for $0 \leq t \leq 2\pi$

$$= \int_0^{2\pi} [-\cos t \sin^2 t + \sin^2 t \cos t]\, dt$$

$$= \int_0^{2\pi} 0\, dt = \boxed{0} \text{ ; and}$$

$R: \begin{cases} 0 \leq \theta \leq 2\pi \\ 0 \leq r \leq 1 \end{cases}$

$$\iint_R (N_x - M_y)\, dA = \iint_R (0 - x)\, dA$$

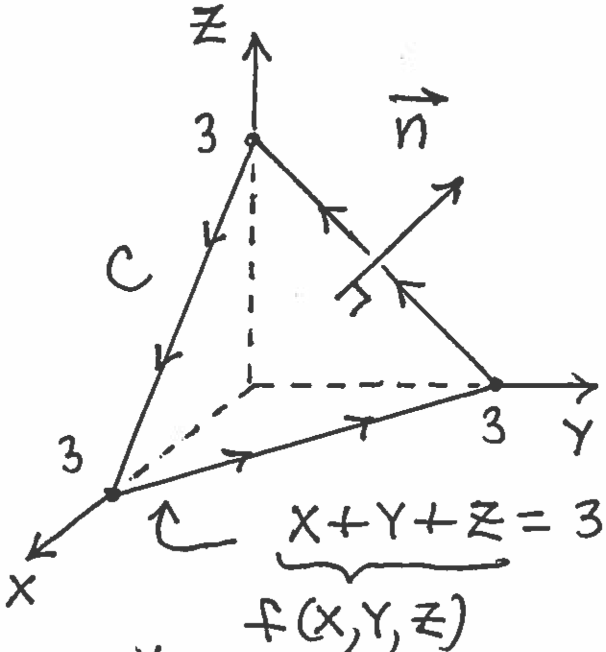
$$= \int_0^{2\pi} \int_0^1 -r\cos\theta \cdot r\, dr\, d\theta$$

$$= \int_0^{2\pi} \cos\theta \left(-\tfrac{1}{3} r^3\right)\Big|_{r=0}^{r=1} d\theta$$

$$= -\tfrac{1}{3} \sin\theta \Big|_0^{2\pi} = -\tfrac{1}{3} \underbrace{\cancel{\sin}(2\pi)}_{0} - \tfrac{-1}{3} \underbrace{\cancel{\sin}(0)}_{0}$$

$= \boxed{0}$, so VERIFIED.

8.) (12 pts.) Find the Flux of the Vector Field $\vec{F}(x,y,z) = (z)\vec{i} + (x)\vec{j} + (y)\vec{k}$ through Surface S, which is that portion of the plane $x+y+z=3$ is the 1st octant, and $\vec{n}$ is the unit normal vector pointing away from the origin.



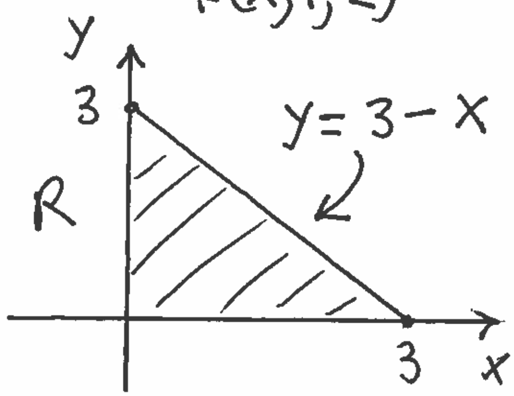
$$\vec{\nabla}f = (1)\vec{i} + (1)\vec{j} + (1)\vec{k} \longrightarrow$$

$$|\vec{\nabla}f| = \sqrt{1^2+1^2+1^2} = \sqrt{3}, \text{ so}$$

$$\vec{n} = \frac{\vec{\nabla}f}{|\vec{\nabla}f|} = \frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k};$$

then

$$\text{Flux} = \iint_S \vec{F}\cdot\vec{n}\, dS$$



$$= \iint_R (\vec{F}\cdot n)\frac{|\vec{\nabla}f|}{|f_z|}\, dS$$

$$= \iint_R \left(\vec{F}\cdot\frac{\vec{\nabla}f}{\cancel{|\vec{\nabla}f|}}\right)\frac{\cancel{|\vec{\nabla}f|}}{\underbrace{|f_z|}_{1}}\, dS$$

$$= \int_0^3\int_0^{3-X} (Z+X+Y)\, dY\, dX$$

$$= \int_0^3\int_0^{3-X} \left((3-\cancel{X}-\cancel{Y})+\cancel{X}+\cancel{Y}\right) dY\, dX$$

$$= \int_0^3 \left(3Y\Big|_{Y=0}^{Y=3-X}\right) dX = \int_0^3 (9-3X)\, dX$$

$$= \left(9X - \frac{3}{2}X^2\right)\Big|_0^3 = 27 - \frac{27}{2} = \boxed{\frac{27}{2}}.$$

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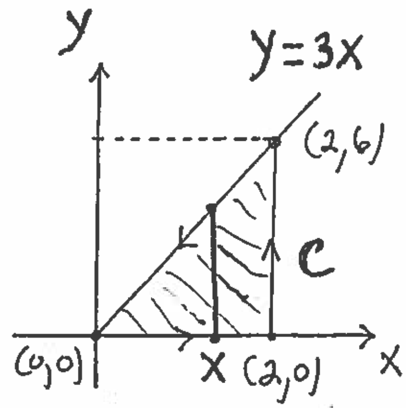
9.) (12 pts.) Let loop C be the triangle with vertices $(0,0)$, $(2,0)$, and $(2,6)$. Evaluate the line integral $\oint xy\,dx + (x-y)\,dy$ using one of Green's Theorems.

$$\underset{\uparrow}{}\ M \qquad \underset{\uparrow}{}\ N \quad \text{so}$$



$$\vec{F}(x,y) = (xy)\vec{i} + (x-y)\vec{j}$$

Green's Theorem 2:

$$\oint xy\,dx + (x-y)\,dy = \oint M\,dx + N\,dy$$

$$= \oint_C \vec{F}\cdot\vec{T}\,ds = \iint_R \left(N_x - M_y\right) dA$$

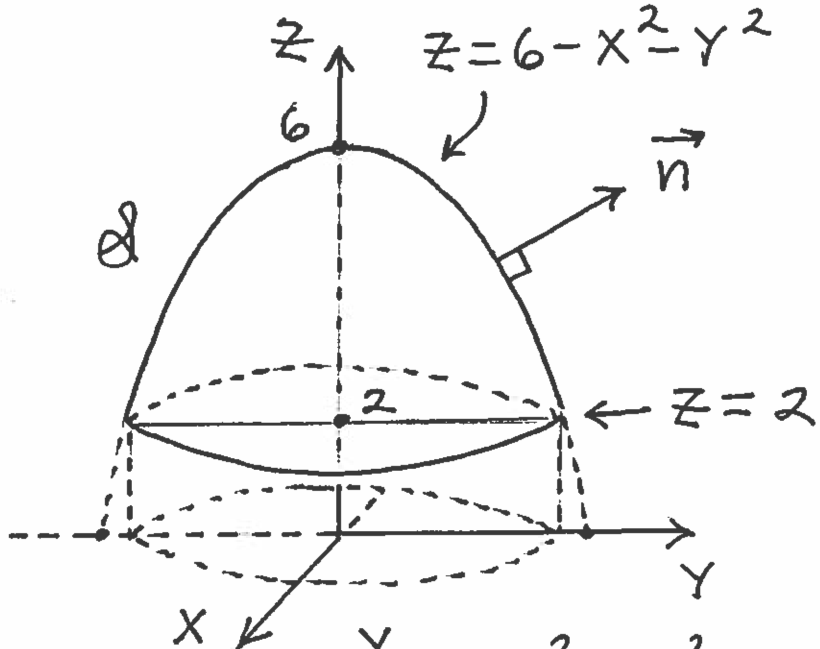
$$= \iint_R (1-x)\,dA = \int_0^2 \int_0^{3x} (1-x)\,dy\,dx$$

$$= \int_0^2 (1-x)\,y \Big|_{y=0}^{y=3x} dx = \int_0^2 (1-x)3x\,dx$$

$$= \int_0^2 (3x-3x^2)\,dx = \left(\tfrac{3}{2}x^2 - x^3\right)\Big|_0^2$$

$$= 6 - 8 = -2$$

10.) (19 pts.) Verify Stoke's Theorem for the Vector Field $\vec{F}(x,y,z) = (-y)\vec{i} + (x)\vec{j} + (z)\vec{k}$, where Surface S is that portion of the paraboloid $z = 6 - x^2 - y^2$, which lies above the plane $z = 2$.



$$6 - x^2 - y^2 = 2 \rightarrow$$

$$x^2 + y^2 = 4 \; ;$$

$$\underbrace{x^2 + y^2 + z}_{f(x,y,z)} = 6$$

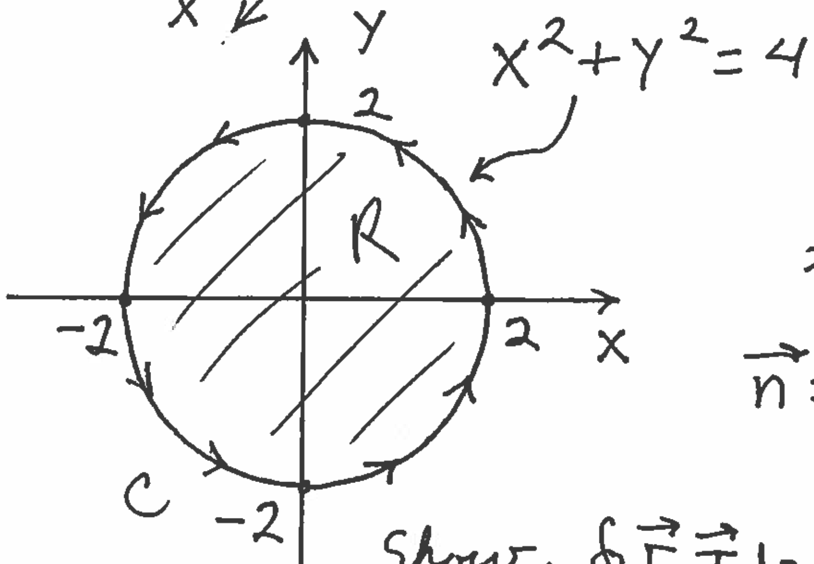
$$\vec{\nabla} f = (2x)\vec{i} + (2y)\vec{j} + (1)\vec{k} \, ,$$

$$|\vec{\nabla} f| = \sqrt{(2x)^2 + (2y)^2 + (1)^2}$$

$$= \sqrt{4x^2 + 4y^2 + 1} \; ;$$

then

$$\vec{n} = \frac{(2x)\vec{i} + (2y)\vec{j} + (1)\vec{k}}{\sqrt{4(x^2+y^2)+1}} \; ;$$



<u>Show</u>: $\oint_C \vec{F} \cdot \vec{T} \, ds = \iint_S \vec{\nabla} \times \vec{F} \cdot \vec{n} \, dS$;

I.) $\oint_C \vec{F} \cdot \vec{T} \, ds = \oint_C M\,dx + N\,dy + P\,dz$

$$C : \begin{cases} x = 2\cos t \\ y = 2\sin t \end{cases} \quad \text{for } 0 \le t \le 2\pi$$

$$= \int_C \left[(-y)(-2\sin t)+(x)(2\cos t)+(z)(0)\right] dt$$

$$= \int_0^{2\pi}\left[(-2\sin t)(-2\sin t)+(2\cos t)(2\cos t)\right] dt$$

$$= \int_0^{2\pi} 4(\sin^2 t+\cos^2 t)\, dt = \int_0^{2\pi} 4\, dt$$

$$= 4t \Big|_0^{2\pi} = \boxed{8\pi} \ ; \text{ then}$$

II.)

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & z \end{vmatrix} = (0-0)\,\vec{i} - (0-0)\,\vec{j} + (1-(-1))\,\vec{k} = 2\vec{k},$$

$$\iint_S \vec{\nabla}\times\vec{F}\cdot\vec{n}\, dS = \iint_R \left(\vec{\nabla}\times\vec{F}\cdot\frac{\vec{\nabla}f}{\cancel{|\vec{\nabla}f|}}\right)\frac{\cancel{|\vec{\nabla}f|}}{\underbrace{|f_z|}_{1}}\, dA$$

$$= \iint_R 2\, dA = 2\underbrace{\iint_R 1\, dA}_{\text{area } R}$$

$$= 2\left(\pi(2)^2\right)$$

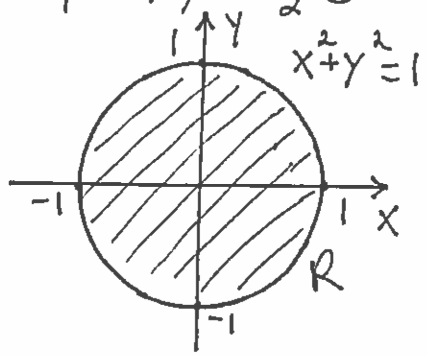
$$= \boxed{8\pi}\ , \text{ so VERIFIED}$$

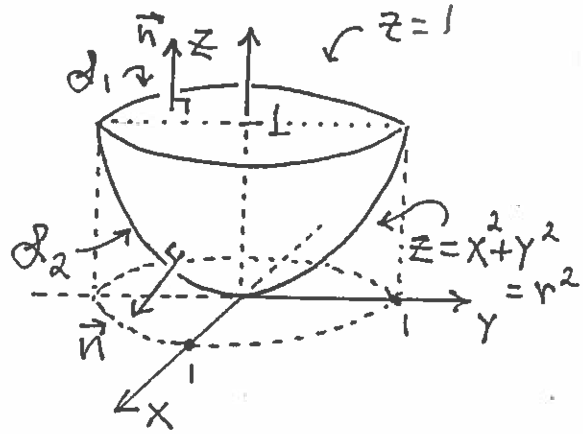
11.) (19 pts.) Verify the Divergence Theorem for $\vec{F}(x, y, z) = (y)\ \vec{i} + (-x)\ \vec{j} + (2z)\ \vec{k}$, where the solid D is enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$.

solid: D

surface: $S = S_1 \text{ (top)} \cup S_2 \text{ (side)}$

$$\text{div } \vec{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} = (0) + (0) + (2) = 2$$





Divergence Theo.: $\iint_S \vec{F} \cdot \vec{n}\, dS = \iiint_D \text{div. } \vec{F}\, dV$

$$D: \begin{cases} 0 \le \theta \le 2\pi \\ 0 \le r \le 1 \\ r^2 \le z \le 1 \end{cases}$$

I.) $= \iiint_D 2\, dV$

$$= \int_0^{2\pi} \int_0^1 \int_{r^2}^1 2 \cdot r\, dz\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^1 \left(2rz \Big|_{z=r^2}^{z=1} \right) dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^1 (2r - 2r^3)\, dr\, d\theta$$

$$= \int_0^{2\pi} \left(r^2 - \frac{1}{2} r^4 \right) \Big|_{r=0}^{1}\, d\theta$$

$$= \int_0^{2\pi} \left(1 - \frac{1}{2} \right) d\theta = \frac{1}{2} \cdot \theta \Big|_0^{2\pi} = \boxed{\pi};$$

II.) $\iint_{S} \vec{F}\cdot\vec{n}\,dS = \iint_{S_1} \vec{F}\cdot\vec{n}\,dS + \iint_{S_2} \vec{F}\cdot\vec{n}\,dS$;

$$\iint_{S_1} \vec{F}\cdot\vec{n}\,dS = \iint_{S_1} \vec{F}\cdot\vec{k}\,dS = \iint_{S_1} 2z\,dS = \iint_{S_1} 2(1)\,dS$$

$$= 2\iint_{S_1} 1\,dS = 2\left(\pi(1)^2\right) = \boxed{2\pi}$$;

$\iint_{S_2} \vec{F}\cdot\vec{n}\,dS$: $\underbrace{x^2+y^2-z}_{f(x,y,z)} = 0 \longrightarrow$

$\vec{\nabla}f = (2x)\vec{i} + (2y)\vec{j} + (-1)\vec{k}$ and

$$|\vec{\nabla}f| = \sqrt{(2x)^2+(2y)^2+(-1)^2} = \sqrt{4x^2+4y^2+1}$$

$$= \sqrt{4(x^2+y^2)+1} = \sqrt{4r^2+1}$$; then

$$= \iint_{R} \vec{F}\cdot\frac{\vec{\nabla}f}{\cancel{|\vec{\nabla}f|}}\cdot\frac{\cancel{|\vec{\nabla}f|}}{|f_z|}\,dA = \iint_{R} -2z\,dA$$

$$= -2\iint_{R} (x^2+y^2)\,dA = -2\int_0^{2\pi}\int_0^1 (r^2)\cdot r\,dr\,d\theta$$

$$= -2\int_0^{2\pi}\int_0^1 r^3\,dr\,d\theta = -2\int_0^{2\pi}\left(\tfrac{1}{4}r^4\Big|_0^1\right)d\theta$$

$$= -2\cdot\tfrac{1}{4}\cdot\left(\theta\Big|_0^{2\pi}\right) = -\tfrac{1}{2}(2\pi) = \boxed{-\pi}$$; then

$2\pi + (-\pi) = \boxed{\pi}$; so verified.