Summary for Line Integrals

1.) (Work and Path Independence)

$\vec{F} = M\vec{i} + N\vec{j} + P\vec{k}$ is conservative

iff $\vec{F} = \vec{\nabla} f$ for some scalar function f

iff on path C from pt. A to pt. B

$$\text{Work} = \int_C \vec{F}\cdot\vec{T}\,ds = \int_C \vec{\nabla} f\cdot\vec{T}\,ds$$

$$= f(P)\Big|_A^B = f(B) - f(A)$$

(and is therefore path independent)

(This also applies to flow along path C.)

TEST: $\vec{F}$ is conservative *iff*

(3D) $M_y = N_x$, $N_z = P_y$, and $P_x = M_z$

(2D) $M_y = N_x$

2.) (Circulation – Flow on closed path C)

Let $\vec{F} = M\vec{i} + N\vec{j}$ then

$$\text{Circ} = \oint_C \vec{F}\cdot\vec{T}\,ds = \int_C M\,dx + N\,dy = \iint_R \underbrace{\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}_{\text{Curl for } \vec{F}} dA$$

Green's Theorem

(This also applies to work along path C.)

3.) (Flux on closed path C)

Let $\vec{F} = M\vec{i} + N\vec{j}$ then

$$\text{Flux} = \oint_C \vec{F}\cdot\vec{n}\,ds = \int_C (M\,dy - N\,dx = \iint_R \underbrace{\left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right)}_{\text{Divergence for } \vec{F}} dA$$

Green's Theorem