Math 21D
Kouba
Applications of Double Integrals

Let \( R \) be a flat region in two-dimensional space and let \( \delta(P) \) be the density of the region at point \( P = (x, y) \).

1.) **AREA**: \( \int_R 1 \, dA \) represents the *area* of region \( R \).

2.) **AVERAGE VALUE**: \( \frac{1}{\text{Area of } R} \int_R f(x, y) \, dA \) represents the *average value* of function \( z = f(x, y) \) over region \( R \).

3.) **MASS**: \( \int_R \delta(P) \, dA \) represents the *mass* of region \( R \).

4.) **VOLUME**: \( \int_R f(P) \, dA \) represents the *volume* of the solid region defined on region \( R \) with height \( f(P) \) at point \( P \).

5.) **MOMENT**:
   a.) \( \int_R (x - a)\delta(P) \, dA \) represents the *moment* of region \( R \) about the vertical line \( x = a \).
   b.) \( \int_R (y - b)\delta(P) \, dA \) represents the *moment* of region \( R \) about the horizontal line \( y = b \).

6.) **CENTER OF MASS, \((\bar{x}, \bar{y})\)**:
   a.) \( \bar{x} = \frac{\int_R x\delta(P) \, dA}{\int_R \delta(P) \, dA} \) represents the *x-coordinate* of the center of mass of region \( R \).
   b.) \( \bar{y} = \frac{\int_R y\delta(P) \, dA}{\int_R \delta(P) \, dA} \) represents the *y-coordinate* of the center of mass of region \( R \).

7.) **CENTROID, \((\bar{x}, \bar{y})\)**:
   a.) \( \bar{x} = \frac{\int_R x \, dA}{\int_R 1 \, dA} \) represents the *x-coordinate* of the centroid of region \( R \).
   b.) \( \bar{y} = \frac{\int_R y \, dA}{\int_R 1 \, dA} \) represents the *y-coordinate* of the centroid of region \( R \).

**NOTE**: The formulas for centroid follow immediately from the formulas for center of mass by letting density \( \delta(P) = 1 \).

8.) **MOMENT OF INERTIA**: \( \int_R (\text{distance})^2 \delta(P) \, dA \) represents the *moment of inertia* of region \( R \), where *distance* refers to the distance from point \( P = (x, y) \) in region \( R \) to either a point or axis (line) of rotation.