Section 15.1
Thomas Calculus
11 th Ed.

Switching the Order of Integration of a Double Integral

The following integrals are nonelementary integrals (the functions has no closed form anti-derivative):

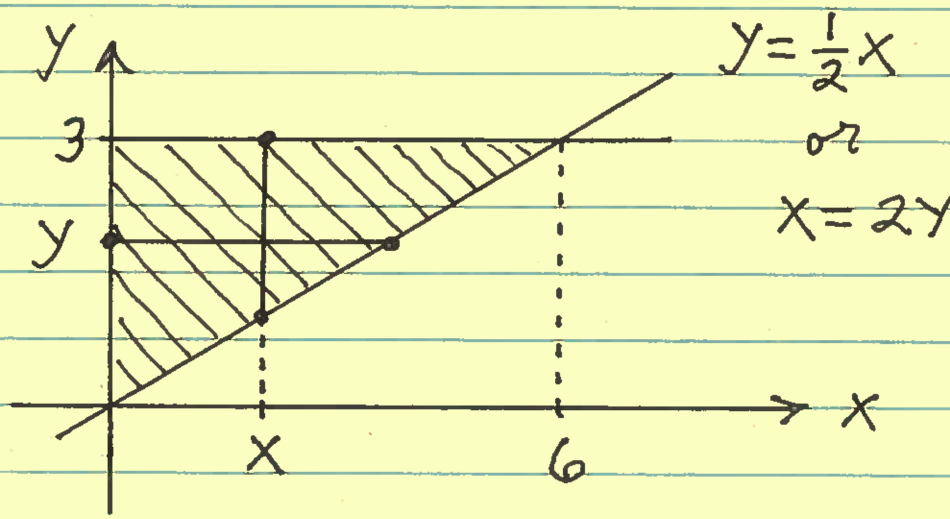
$$\int e^{x^2}\,dx \quad , \quad \int \cos(y^2)\,dy,$$
$$\int \frac{\sin x}{x}\,dx \quad , \quad \int \sqrt{1+y^4}\,dy$$

If you encounter these functions when doing Double Integrals, you will need to Switch the Order of Integration. This will require you to graph the region R of integration and using

Vertical and Horizontal Cross-Sections.

<u>Example</u>: Evaluate each Double Integral by first Switching the Order of Integration.

1.) $\int_0^6 \int_{\frac{1}{2}x}^3 \cos(y^2)\, dy\, dx$



$$R: \begin{cases} 0 \le x \le 6 \\ \frac{1}{2}x \le y \le 3 \end{cases} \quad \text{and} \quad \begin{cases} 0 \le y \le 3 \\ 0 \le x \le 2y \end{cases}$$

(Now Switch Order)

$$= \int_0^3 \int_0^{2y} \cos(y^2)\, dx\, dy$$

$$= \int_0^3 \cos(y^2) \cdot x \Big|_{x=0}^{x=2y} dy$$

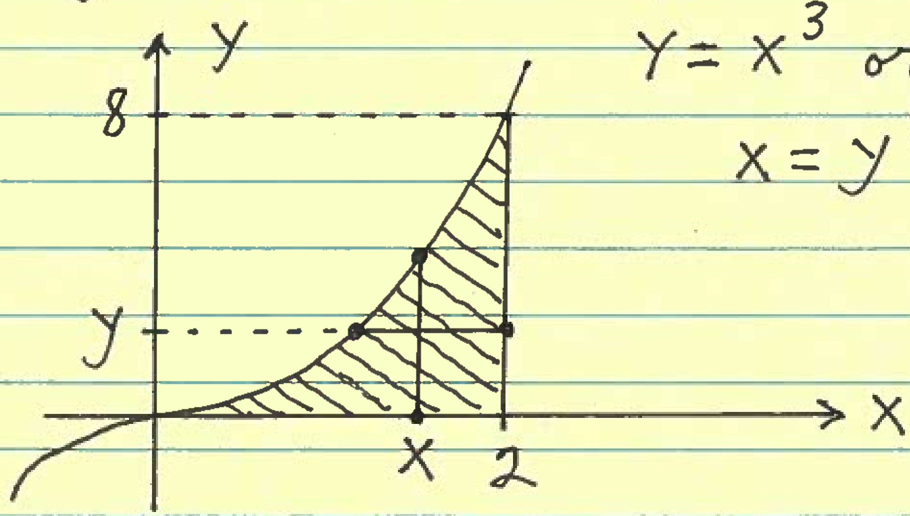
$$= \int_0^3 \left(\cos(y^2) \cdot (2y) - \cos(y^2)(0) \right) dy$$

$$= \int_0^3 2y \cos(y^2)\, dy$$

$$= \sin(y^2) \Big|_0^3 = \sin 9 - \overset{0}{\cancel{\sin 0}}$$

$$= \sin 9$$

2.) $\displaystyle \int_0^8 \int_{y^{1/3}}^2 \sqrt{1+x^4}\, dx\, dy$



$Y = x^3$ or

$x = y^{1/3}$

$$R: \begin{cases} 0 \le y \le 8 \\ y^{1/3} \le x \le 2 \end{cases} \quad \text{and} \quad \begin{cases} 0 \le x \le 2 \\ 0 \le y \le x^3 \end{cases}$$

(Now Switch Order)

$$= \int_0^2 \int_0^{x^3} \sqrt{1+x^4}\, dy\, dx$$

$$= \int_0^2 \sqrt{1+x^4} \cdot y \Big|_{y=0}^{y=x^3} dx$$

$$= \int_0^2 \left(\sqrt{1+x^4} \cdot (x^3) - \sqrt{1+x^4} \cdot (0) \right) dx$$

$$= \int_0^2 x^3 \sqrt{1+x^4}\, dx$$

$$= \frac{1}{4} \cdot \frac{2}{3} (1+x^4)^{3/2} \Big|_0^2$$

$$= \frac{1}{6}(17)^{3/2} - \frac{1}{6}(1)^{3/2}$$

$$= \frac{1}{6}(17)^{3/2} - \frac{1}{6}$$