

Math 21D

Kouba

Applications of Triple Integrals

Let R be a solid region in three-dimensional space and let $\delta(P)$ be the density of the region at point $P = (x, y, z)$.

- 1.) VOLUME : $\int_R 1 dV$ represents the *volume* of region R .
- 2.) AVERAGE VALUE : $\frac{1}{\text{Volume of } R} \int_R f(x, y, z) dV$ represents the *average value* of function $w = f(x, y, z)$ over region R .
- 3.) MASS : $\int_R \delta(P) dV$ represents the *mass* of region R .
- 4.) MOMENT :
 - a.) $\int_R (x - a)\delta(P) dV$ represents the *moment* of region R about the plane $x = a$.
 - b.) $\int_R (y - b)\delta(P) dV$ represents the *moment* of region R about the plane $y = b$.
 - c.) $\int_R (z - c)\delta(P) dV$ represents the *moment* of region R about the plane $z = c$.
- 5.) CENTER OF MASS, $(\bar{x}, \bar{y}, \bar{z})$:
 - a.) $\bar{x} = \frac{\int_R x\delta(P) dV}{\int_R \delta(P) dV}$ represents the *x-coordinate* of the center of mass of region R .
 - b.) $\bar{y} = \frac{\int_R y\delta(P) dV}{\int_R \delta(P) dV}$ represents the *y-coordinate* of the center of mass of region R .
 - c.) $\bar{z} = \frac{\int_R z\delta(P) dV}{\int_R \delta(P) dV}$ represents the *z-coordinate* of the center of mass of region R .
- 6.) CENTROID, $(\bar{x}, \bar{y}, \bar{z})$:
 - a.) $\bar{x} = \frac{\int_R x dV}{\int_R 1 dV}$ represents the *x-coordinate* of the centroid of region R .
 - b.) $\bar{y} = \frac{\int_R y dV}{\int_R 1 dV}$ represents the *y-coordinate* of the centroid of region R .

c.) $\bar{z} = \frac{\int_R z \, dV}{\int_R 1 \, dV}$ represents the *z-coordinate* of the center of mass of region R .

NOTE : The formulas for centroid follow immediately from the formulas for center of mass by letting density $\delta(P) = 1$.

7.) MOMENT OF INERTIA : $\int_R (\text{distance})^2 \delta(P) \, dV$ represents the *moment of inertia* of region R , where *distance* refers to the distance from point $P = (x, y, z)$ in region R to either a point or axis (line) of rotation.