Math 21D
Kouba
Triple Integrals Over Solid Regions R
Using Rectangular Coordinates

Assume that function $z = f(P)$ is defined on a solid region R in 3D-space, i.e., $z = f(x, y, z)$. Partition R into n parts $R_1, R_2, \ldots, R_n$ of volumes $\Delta V_1, \Delta V_2$, and ΔV_n, resp. Pick sampling points $P_i = (x_i, y_i, z_i)$ in R_i for $i = 1, 2, \ldots, n$. Define the diameter of R_i, $\mathrm{diam}(R_i)$, to be the maximum distance between points in R_i and define the mesh of the partition to be

$$\text{mesh} = \max_{1 \le i \le n} (\mathrm{diam}(R_i)) .$$

Then

$$\iiint_R f(P)\, dV = \lim_{\text{mesh} \to 0} \sum_{i=1}^{n} f(P_i) \cdot \Delta V_i .$$

If $\Delta V_i = (\Delta z_i)(\Delta y_i)(\Delta x_i)$ for $i = 1, 2, \ldots, n$, then

$$\iiint_R f(P)\, dV = \iiint_R f(P)\, dz\, dy\, dx .$$

<u>Facts</u>: 1.) $\iiint_R 1 \, dV =$ Volume of R

2.) $\iiint_R \delta(P) \, dV =$ Mass of R, where $\delta(P)$ is density $\left(\frac{\text{mass}}{\text{volume}} \text{ units}\right)$

3.) The <u>average</u> <u>value</u> of $z = f(P)$ on solid region R is

$$AVE = \frac{1}{\text{volume of } R} \cdot \iiint_R f(P) \, dV$$