Example: Consider an object of mass $m = 3$ gm. moving along path C, which is determined by the vector function

$$\vec{r}(t) = (t^2-1)\,\vec{i} + \left(\frac{1}{3}t^3\right)\vec{j}\ ,$$

for $t \geq 0$, and t is seconds and distance is cm.

1.) Find and sketch path C.

2.) Find the velocity vector $\vec{v}(t)$ and the acceleration vector $\vec{a}(t)$.

3.) Find a formula for the speed of motion at time t.

4.) Find a formula for the acceleration of motion at time t.

5.) Find and plot $\vec{r}(1)$, $\vec{v}(1)$, and $\vec{a}(1)$. Find the speed and acceleration of motion when $t = 1$ sec.

6.) Find and plot $\vec{r}(2)$, $\vec{v}(2)$, and $\vec{a}(2)$. Find the speed and acceleration of motion when $t = 2$ sec.

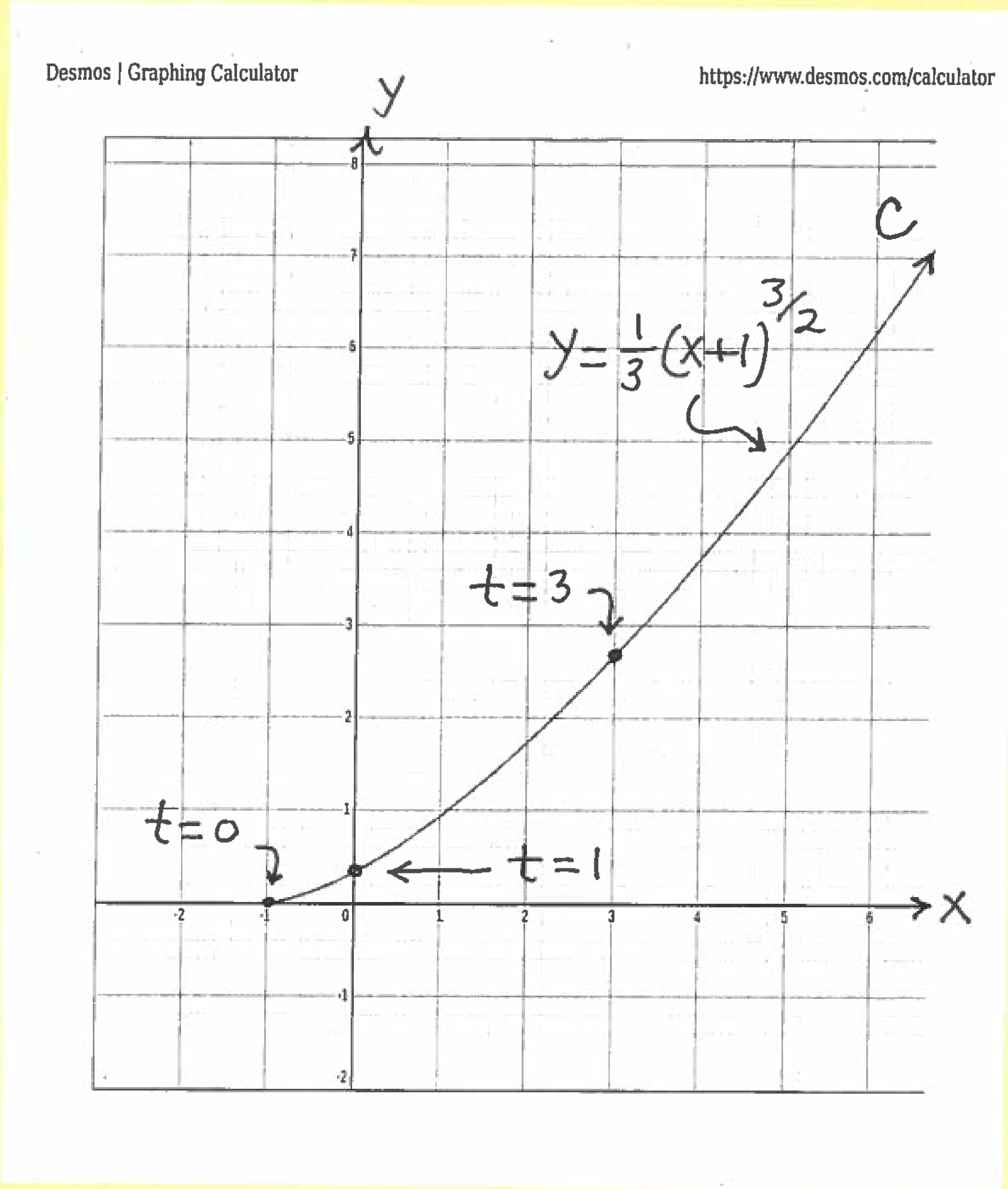
7.) Find the force vector which changes the speed and direction of the object's motion when $t=1$ sec. Find the magnitude of this force.

1.) $\begin{cases} x = t^2 - 1 \\ y = \frac{1}{3}t^3 \end{cases} \rightarrow 3y = t^3 \rightarrow$

$t = (3y)^{1/3} \rightarrow$ (SUB) $\rightarrow$

$x = \left((3y)^{1/3}\right)^2 - 1 \rightarrow x+1 = (3y)^{2/3} \rightarrow$

$3y = (x+1)^{3/2} \rightarrow \boxed{y = \frac{1}{3}(x+1)^{3/2}}$.



2.) $\vec{r}(t) = (t^2-1)\vec{i} + \left(\frac{1}{3}t^3\right)\vec{j} \xrightarrow{D}$

$\vec{v}(t) = (2t)\vec{i} + (t^2)\vec{j} \xrightarrow{D}$

$\vec{a}(t) = (2)\vec{i} + (2t)\vec{j}$;

3.) speed $\dfrac{ds}{dt} = |\vec{v}(t)|$

$$= \sqrt{(2t)^2 + (t^2)^2} = \sqrt{4t^2 + t^4} \text{, i.e.,}$$

$$\frac{ds}{dt} = \sqrt{4t^2 + t^4} .$$

4.) acceleration

$$\frac{d^2 s}{dt^2} = \frac{d}{dt} \sqrt{4t^2 + t^4}$$

$$= \frac{1}{2}(4t^2 + t^4)^{-1/2} \cdot (8t + 4t^3)$$

$$= \frac{4t + 2t^3}{\sqrt{4t^2 + t^4}} \text{, i.e.,}$$

$$\frac{d^2 s}{dt^2} = \frac{4t + 2t^3}{\sqrt{4t^2 + t^4}} .$$

5.) $\vec{r}(1) = (0)\vec{i} + \left(\frac{1}{3}\right)\vec{j}$,
$\vec{v}(1) = (2)\vec{i} + (1)\vec{j}$,
$\vec{a}(1) = (2)\vec{i} + (2)\vec{j}$,

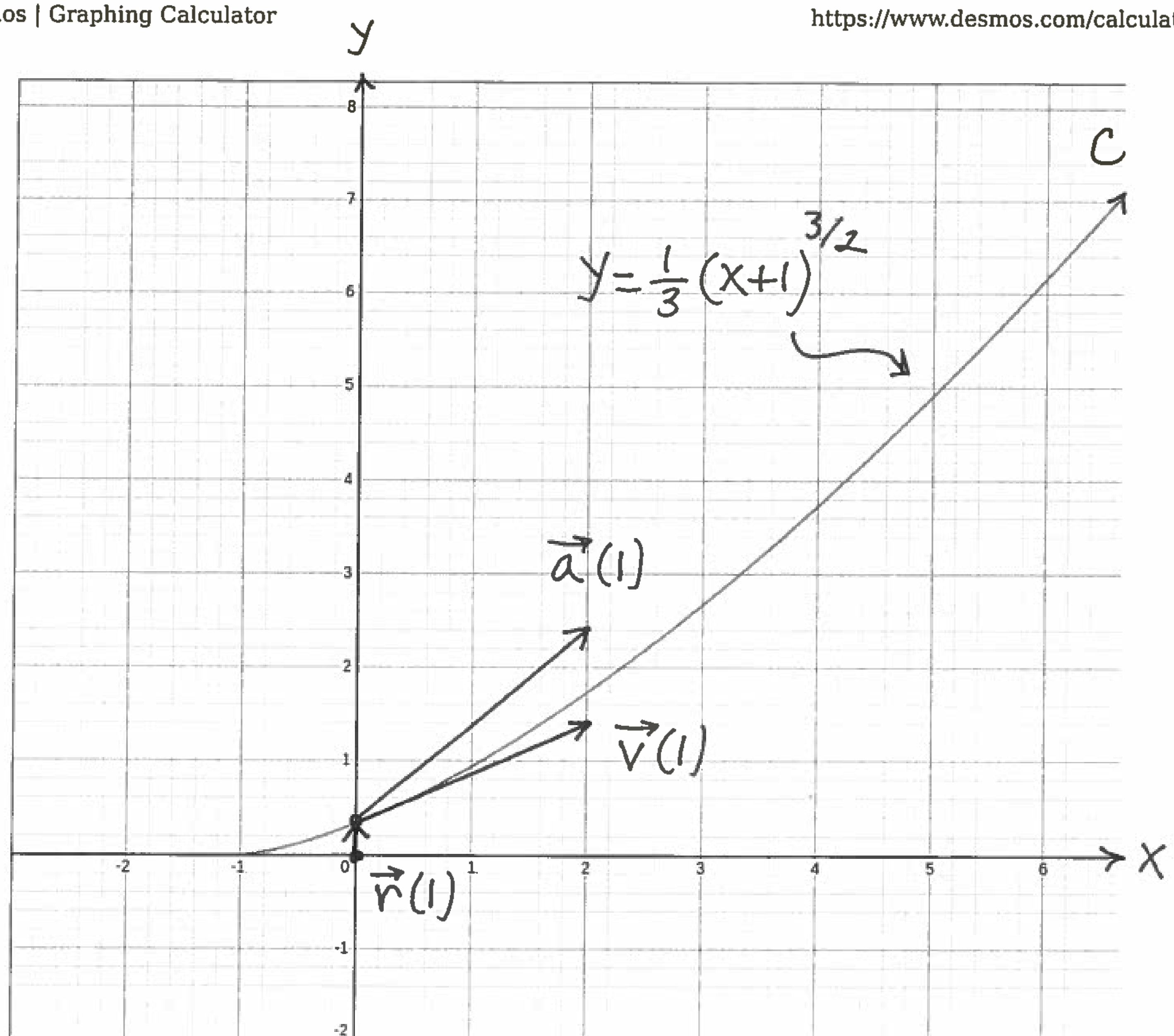
speed at $t = 1$ sec. is

$$\frac{ds}{dt} = \sqrt{4(1) + (1)^4} = \sqrt{5} \approx 2.24 \text{ cm./sec.};$$

acceleration at $t = 1$ sec. is

$$\frac{d^2s}{dt^2} = \frac{4(1) + 2(1)^3}{\sqrt{4(1)^2 + (1)^4}} = \frac{6}{\sqrt{5}} \approx 2.68 \text{ cm./sec.}^2$$

Desmos | Graphing Calculator https://www.desmos.com/calculator



6.) $\vec{r}(2) = (3)\vec{i} + \left(\frac{8}{3}\right)\vec{j}$,

$\vec{v}(2) = (4)\vec{i} + (4)\vec{j}$,

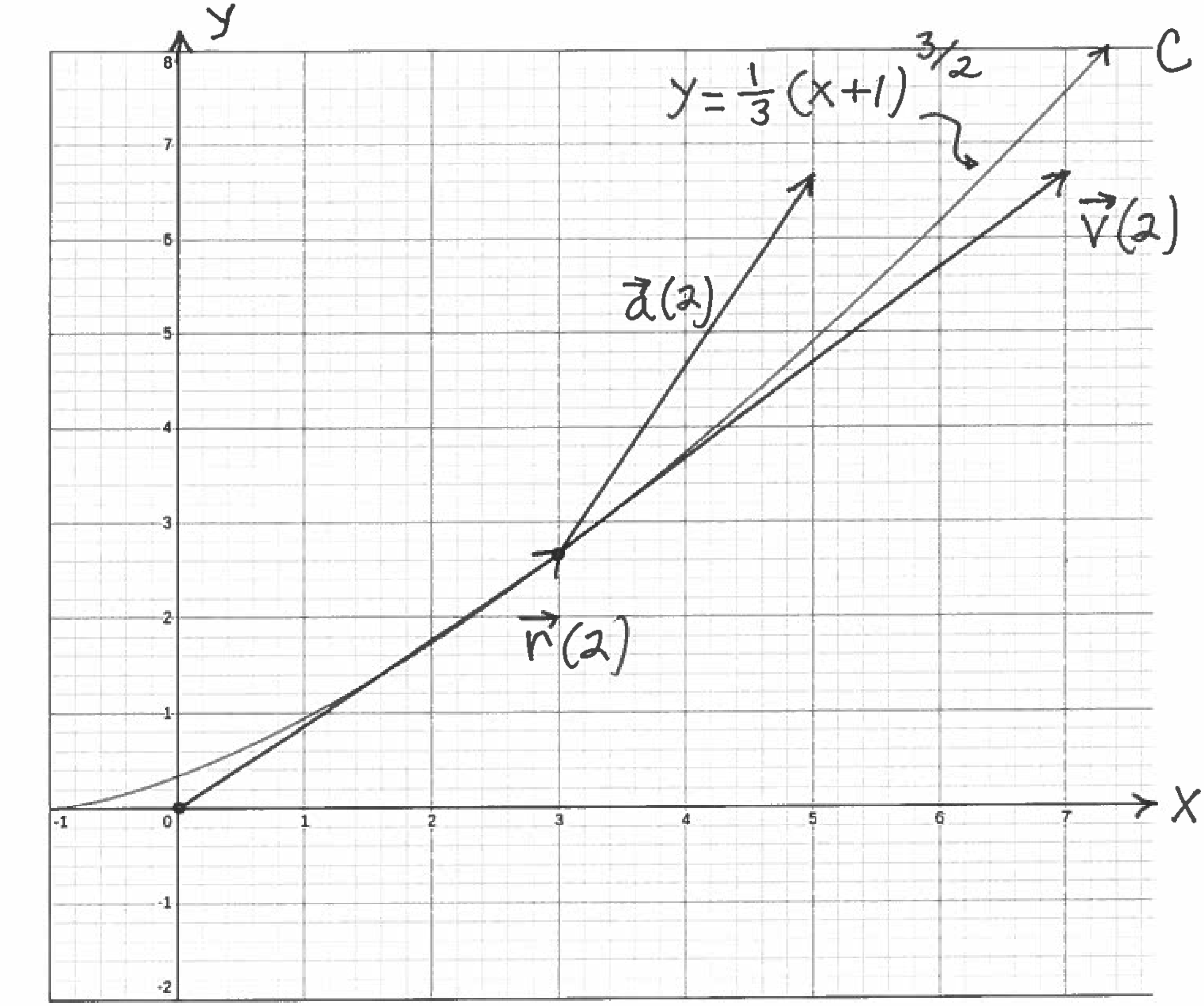
$\vec{a}(2) = (2)\vec{i} + (4)\vec{j}$;

speed at $t = 2$ sec. is

$$\frac{ds}{dt} = \sqrt{4(2)^2 + (2)^4}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2} \approx 5.66 \text{ cm./sec.};$$

acceleration at $t = 2$ sec. is

$$\frac{d^2s}{dt^2} = \frac{4(2) + 2(2)^3}{\sqrt{4(2)^2 + (2)^4}}$$
$$= \frac{24}{\sqrt{32}} = \frac{24}{4\sqrt{2}}$$
$$= \frac{6}{\sqrt{2}} \approx 4.24 \text{ cm./sec.}^2 .$$



$y = \left(\frac{1}{3}\right)(x+1)^{1.5}$

7.) When $t = 1$ sec. the acceleration vector is $\vec{a}(t) = (2)\vec{i} + (2t)\vec{j}$, so $\vec{a}(1) = 2\vec{i} + 2\vec{j}$. The mass $m = 3$ gm., so the force vector is

$$\begin{aligned}\vec{F}(1) &= m \cdot \vec{a}(1) \\ &= (3)(2\vec{i} + 2\vec{j}) \\ &= 6\vec{i} + 6\vec{j}, \text{ i.e.,}\end{aligned}$$

$$\vec{F}(1) = 6\vec{i} + 6\vec{j} \quad ;$$

the magnitude of this force vector is

$$\begin{aligned}|\vec{F}(1)| &= \sqrt{(6)^2 + (6)^2} \\ &= \sqrt{72} \\ &= 6\sqrt{2} \\ &\approx 8.48 \text{ dynes}.\end{aligned}$$