Math22A
Kouba
Zero Matrices, Matrix Arithmetic, and Transpose Arithmetic

THEOREM 1.4.2 Properties of Zero Matrices

If c is a scalar, and if the sizes of the matrices are such that the operations can be perfomed, then:

(a) $A + 0 = 0 + A = A$

(b) $A - 0 = A$

(c) $A - A = A + (-A) = 0$

(d) $0A = 0$

(e) *If* $cA = 0$, *then* $c = 0$ *or* $A = 0$.

THEOREM 1.4.1 Properties of Matrix Arithmetic

Assuming that the sizes of the matrices are such that the indicated operations can performed, the following rules of matrix arithmetic are valid.

(a) $A + B = B + A$ **[Commutative law for matrix addition]**

(b) $A + (B + C) = (A + B) + C$ **[Associative law for matrix addition]**

(c) $A(BC) = (AB)C$ **[Associative law for matrix multiplication]**

(d) $A(B + C) = AB + AC$ **[Left distributive law]**

(e) $(B + C)A = BA + CA$ **[Right distributive law]**

(f) $A(B - C) = AB - AC$

(g) $(B - C)A = BA - CA$

(h) $a(B + C) = aB + aC$

(i) $a(B - C) = aB - aC$

(j) $(a + b)C = aC + bC$

(k) $(a - b)C = aC - bC$

(l) $a(bC) = (ab)C$

(m) $a(BC) = (aB)C = B(aC)$

THEOREM 1.4.8 *If the sizes of the matrices are such that the stated operations can be performed, then:*

(a) $(A^T)^T = A$

(b) $(A + B)^T = A^T + B^T$

(c) $(A - B)^T = A^T - B^T$

(d) $(kA)^T = kA^T$

(e) $(AB)^T = B^TA^T$