Section 1.1

1.) a.) Yes b.) No, because of $x_1 x_3$
c.) Yes d.) No, because of x_1^{-2}
e.) No, because of $x_1^{3/5}$ f.) Yes

5.) a.) $\begin{cases} 2x_1 + 0\cdot x_2 = 0 \\ 3x_1 + {-4}x_2 = 0 \\ 0\cdot x_1 + 1\cdot x_2 = 1 \end{cases}$ b.) $\begin{cases} 3x_1 + 0\cdot x_2 - 2x_3 = 5 \\ 7x_1 + 1\cdot x_2 + 4x_3 = -3 \\ 0\cdot x_1 - 2x_2 + 1\cdot x_3 = 7 \end{cases}$

7.) a.) $\left[\begin{array}{c|c} -2 & 6 \\ 3 & 8 \\ 9 & -3 \end{array}\right]$ b.) $\left[\begin{array}{ccc|c} 6 & -1 & 3 & 4 \\ 0 & 5 & -1 & 1 \end{array}\right]$

c.) $\left[\begin{array}{ccccc|c} 0 & 2 & 0 & -3 & -1 & 0 \\ -3 & -1 & 1 & 0 & 0 & -1 \\ 6 & 2 & -1 & 2 & -3 & 6 \end{array}\right]$

8.) a.) $\left[\begin{array}{cc|c} 3 & -2 & -1 \\ 4 & 5 & 3 \\ 7 & 3 & 2 \end{array}\right]$ b.) $\left[\begin{array}{ccc|c} 2 & 0 & 2 & 1 \\ 3 & -1 & 4 & 7 \\ 6 & 1 & -1 & 0 \end{array}\right]$

c.) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

9.) a.) $\begin{cases} 2(3) - 4(1) - 1 = 1 \\ 3 - 3(1) + 1 = 1 \\ 3(3) - 5(1) - 3(1) = 1 \end{cases}$ (YES) b.) $2(3) - 4(-1) - 1 \neq 1$ (NO)

c) $\cancel{2(13) - 4(5) - 2 \neq 1}$ ~~(NO)~~

d.) $\begin{cases} \cancel{2\left(\frac{13}{2}\right) - 4\left(\frac{5}{2}\right) - 2 = 1} \\ \frac{13}{2} - 3\left(\frac{5}{2}\right) + 2 = 1 \\ 3\left(\frac{13}{2}\right) - 5\left(\frac{5}{2}\right) - 3(2) = 1 \end{cases}$ (YES)

11.) a.) $\begin{cases} 3X - 2Y = 4 \\ 6X - 4Y = 9 \end{cases}$ $\to -6X + 4Y = -8$ (ADD) $\to 0 = 1$

so NO SOLUTION

b.) $\begin{cases} 2X - 4Y = 1 \\ 4X - 8Y = 2 \end{cases}$ $\to -4X + 8Y = -2$ (ADD) $\to 0 = 0$

so INFINITELY many solutions:

$2X - 4Y = 1$, let $y = t$ any # $\to$

$2X = 4t + 1 \to X = 2t + 1/2$

c.) $\begin{cases} X - 2Y = 0 \\ X - 4Y = 8 \end{cases}$ $\to X = 2Y$ (SUB) $\to$

$(2Y) - 4Y = 8 \to -2Y = 8 \to$

$Y = -4$, $X = -8$ so UNIQUE SOLUTION

12.) $\begin{cases} 2X - 3Y = a \\ 4X - 6Y = b \end{cases}$ $\to -4X + 6Y = -2a$ (ADD) $\to$

$0 = b - 2a$; if $b - 2a = 0 \rightarrow b = 2a$, then INFINITELY many solutions; if $b - 2a \neq 0 \rightarrow b \neq 2a$, then NO SOLUTION

14.) a.) $x + 10y = 2$: let $y = t$ any # $\rightarrow$
$x = 2 - 10t$

b.) $x_1 + 3x_2 - 12x_3 = 3$: let $x_3 = t$ any #, $x_2 = s$ any # $\rightarrow x_1 = 3 - 3s + 12t$

c.) $4x_1 + 2x_2 + 3x_3 + x_4 = 20$: let $x_4 = t$ any #, $x_3 = s$ any #, $x_2 = r$ any # $\rightarrow$
$4x_1 = 20 - 2r - 3s - t \rightarrow$
$x_1 = 5 - \frac{1}{2}r - \frac{3}{4}s - \frac{1}{4}t$

16.) a.) $\begin{cases} 6x_1 + 2x_2 = -8 \\ 3x_1 + x_2 = -4 \end{cases} \rightarrow -6x_1 - 2x_2 = 8$ (ADD)

$\rightarrow 0 = 0 \rightarrow$ let $x_2 = t$ any # $\rightarrow$
$3x_1 = -t - 4 \rightarrow x_1 = -\frac{1}{3}t - \frac{4}{3}$

b.) $\left[\begin{array}{ccc|c} 2 & -1 & 2 & -4 \\ 6 & -3 & 6 & -12 \\ -4 & 2 & -4 & 8 \end{array}\right]$ $(\times)(-3)(+)$ $(\times)(2)(+)$

$\sim \left[\begin{array}{ccc|c} 2 & -1 & 2 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow 2x_1 - x_2 + 2x_3 = -4 \rightarrow$
let $x_3 = t$ any #, $x_2 = s$ any # $\rightarrow$

$2x_1 = s - 2t - 4 \rightarrow x_1 = \frac{1}{2}s - t - 2$

17.) a.) ADD 2 times the 2nd row to the 1st row.

b.) INTERCHANGE rows 1 and 3.

18.) a.) MULT. row 1 by -3, then add this row to the 2nd row creating a leading 1 in row 2; now interchange rows 1 and 2.

b.) MULT. row 2 by -2, then add this row to the 1st row

19.) a.) $\left[\begin{array}{cc|c} 1 & k & -4 \\ 4 & 8 & 2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & k & -4 \\ 0 & 8-4k & 18 \end{array}\right]$;

if $8-4k=0 \Rightarrow k=2 \Rightarrow$ last row is $\left[\begin{array}{cc|c} 0 & 0 & 18 \end{array}\right] \Rightarrow$ NO SOLUTION;

if $8-4k \neq 0 \Rightarrow \boxed{k \neq 2} \Rightarrow$ last row is $\left[\begin{array}{cc|c} 0 & 8-4k & 18 \end{array}\right]$

$\sim \left[\begin{array}{cc|c} 0 & 1 & 18/(8-4k) \end{array}\right] \Rightarrow$ which leads to a unique solution

b.) $\left[\begin{array}{cc|c} 1 & k & -1 \\ 4 & 8 & -4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & k & -1 \\ 0 & 8-4k & 0 \end{array}\right]$;

if $8-4k=0 \Rightarrow k=2 \Rightarrow$ last row is $[0\ 0 | 0] \Rightarrow$ infinitely many solutions; if $8-4k \neq 0 \Rightarrow k \neq 2 \Rightarrow$ last row is $[0\ \ 8-4k | 0] \sim [0\ \ 1 | 0] \Rightarrow$ which leads to a unique solution; so this system is solvable for all values of k.

20.) a.) $\left[\begin{array}{cc|c} 3 & -4 & k \\ -6 & 8 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 3 & -4 & k \\ 0 & 0 & 5+2k \end{array}\right]$ and

we want $5+2k=0 \rightarrow 2k=-5 \rightarrow$
$k = -5/2$

b.) $\left[\begin{array}{cc|c} k & 1 & -2 \\ 4 & -1 & 2 \end{array}\right] \sim \left[\begin{array}{cc|c} k & 1 & -2 \\ 4+k & 0 & 0 \end{array}\right]$ and we

want $4+k=0 \rightarrow k=-4$ (infinitely many solutions); if $4+k \neq 0$

then $(4+k)x_1 + 0 \cdot x_2 = 0 \rightarrow x_1 = 0 \rightarrow$

$k \cdot x_1 + 1 \cdot x_2 = -2 \rightarrow k \cdot (0) + x_2 = -2 \rightarrow$

$x_2 = -2$

24.) a.) at least 2 of the lines are parallel

b.) the 3 lines intersect at a single point

c.) all 3 lines are multiples of each other

25.) F : 7 units, P : 9 units, C : 16 units,

x : oz. of food 1, y : oz. of food 2, z : oz. of food 3, then

$$\begin{cases} 2x + 3y + z = 7 \\ 2x + y + 3z = 9 \\ 4x + 2y + 5z = 16 \end{cases}$$

26.) $y = ax^2 + bx + c$ and pts. (1,1), (2,4), and (-1,1) : plug pts. into equation $\rightarrow$

$$\begin{cases} a + b + c = 1 \\ 4a + 2b + c = 4 \\ a - b + c = 1 \end{cases} \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 4 & 2 & 1 & 4 \\ 1 & -1 & 1 & 1 \end{array}\right] \begin{array}{l} (x)(-4)(+) \\ \\ (x)(-1)(+) \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -3 & 0 \\ 0 & -2 & 0 & 0 \end{array}\right] (x)(-\tfrac{1}{2}) \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -3 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \begin{array}{l} (x)(-1)(+) \\ \\ (x)(2)(+) \end{array}$$

$$\sim \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 0 & -3 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ (x)(-\tfrac{1}{3}) \\ \\ \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \begin{array}{l} \\ (x)(-1)(+) \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \longrightarrow a=1,\ b=0,\ c=0 \longrightarrow$$

$$y = x^2$$

27.) Let x, y, z be 3 #'s,

$$\begin{cases} x + y + z = 12 \\ 2x + y + 2z = 5 \\ z = x + 1 \end{cases}$$

TRUE/FALSE

(a) T (b) F (c) T (d) T

(e) F (f) F (g) T (h) F