Section 3.2

1.) a.) $\vec{v} = \overrightarrow{(2,2,2)}$ so $\|\vec{v}\| = \sqrt{2^2+2^2+2^2}$

$= \sqrt{12} = \sqrt{4\cdot 3} = 2\sqrt{3}$, then unit vector in same direction is

$\vec{u} = \frac{1}{2\sqrt{3}}\overrightarrow{(2,2,2)} = \overrightarrow{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$;

opposite direction is

$\vec{w} = \overrightarrow{\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)}$

2.) b.) $\vec{v} = \overrightarrow{(-2,3,3,-1)}$ so

$\|\vec{v}\| = \sqrt{(-2)^2+3^2+3^2+(-1)^2} = \sqrt{23}$,

then unit in same direction is

$\vec{u} = \frac{1}{\sqrt{23}}\overrightarrow{(-2,3,3,-1)} = \overrightarrow{\left(\frac{-2}{\sqrt{23}}, \frac{3}{\sqrt{23}}, \frac{3}{\sqrt{23}}, \frac{-1}{\sqrt{23}}\right)}$;

opposite direction is

$\vec{w} = \overrightarrow{\left(\frac{2}{\sqrt{23}}, \frac{-3}{\sqrt{23}}, \frac{-3}{\sqrt{23}}, \frac{1}{\sqrt{23}}\right)}$

3.) a.) $\|\vec{u}+\vec{v}\| = \|\overrightarrow{(2,-2,3)} + \overrightarrow{(1,-3,4)}\|$

$= \|\overrightarrow{(3,-5,7)}\| = \sqrt{3^2+(-5)^2+7^2} = \sqrt{83}$

b.) $\|\vec{u}\| + \|\vec{v}\| = \|\overrightarrow{(2,-2,3)}\| + \|\overrightarrow{(1,-3,4)}\|$

$= \sqrt{2^2+(-2)^2+3^2} + \sqrt{1^2+(-3)^2+4^2} = \sqrt{17} + \sqrt{26}$

c.) $\|-2\vec{u}+2\vec{v}\| = \|-2\overrightarrow{(2,-2,3)} + 2\overrightarrow{(1,-3,4)}\|$

$= \|\overrightarrow{(-4,4,-6)} + \overrightarrow{(2,-6,8)}\| = \|\overrightarrow{(-2,-2,2)}\|$

$= \sqrt{(-2)^2+(-2)^2+2^2} = \sqrt{12} = 2\sqrt{3}$

d.) $\|3\vec{u}-5\vec{v}+\vec{w}\| = \|3\overrightarrow{(2,-2,3)} - 5\overrightarrow{(1,-3,4)} + \overrightarrow{(3,6,-4)}\|$

$= \|\overrightarrow{(6,-6,9)} + \overrightarrow{(-5,15,-20)} + \overrightarrow{(3,6,-4)}\|$

$= \|\overrightarrow{(4,15,-15)}\| = \sqrt{4^2+15^2+(-15)^2}$

$= \sqrt{16+225+225} = \sqrt{466}$

7.) $\vec{v} = \overrightarrow{(-2,3,0,6)}$, if $\|k\vec{v}\| = 5 \Rightarrow$

$\|k\overrightarrow{(-2,3,0,6)}\| = |k| \cdot \|\overrightarrow{(-2,3,0,6)}\|$

$= |k|\sqrt{(-2)^2+3^2+0^2+6^2} = |k|\sqrt{49} = 7|k| = 5$

$\Rightarrow |k| = 5/7 \Rightarrow k = \pm 5/7$

9.) a.) $\vec{u}\cdot\vec{v} = \overrightarrow{(3,1,4)} \cdot \overrightarrow{(2,2,-4)}$

$= 6+2-16 = -8$;

$\vec{u}\cdot\vec{u} = \overrightarrow{(3,1,4)} \cdot \overrightarrow{(3,1,4)} = 9+1+16 = 26$;

$\vec{v}\cdot\vec{v} = \overrightarrow{(2,2,-4)} \cdot \overrightarrow{(2,2,-4)} = 4+4+16 = 24$

b.) $\vec{u}\cdot\vec{v} = \overrightarrow{(1,1,4,6)} \cdot \overrightarrow{(2,-2,3,-2)}$

$= 2-2+12-12 = 0$;

$\vec{u} \cdot \vec{u} = \overrightarrow{(1,1,4,6)} \cdot \overrightarrow{(1,1,4,6)}$

$= 1+1+16+36 = 54$;

$\vec{v} \cdot \vec{v} = \overrightarrow{(2,-2,3,-2)} \cdot \overrightarrow{(2,-2,3,-2)}$

$= 4+4+9+4 = 21$

11.) a.) $d(\vec{u},\vec{v}) = \|\vec{u}-\vec{v}\|$

$= \|\overrightarrow{(3,3,3)} - \overrightarrow{(1,0,4)}\| = \|\overrightarrow{(2,3,-1)}\|$

$= \sqrt{2^2+3^2+(-1)^2} = \sqrt{14}$;

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \dfrac{3+0+12}{\sqrt{3^2+3^2+3^2}\cdot\sqrt{1^2+0^2+4^2}}$

$= \dfrac{15}{\sqrt{27}\,\sqrt{17}} = \dfrac{15}{3\sqrt{3}\,\sqrt{17}} = \dfrac{5}{\sqrt{51}}$, so

θ is acute

b.) $d(\vec{u},\vec{v}) = \|\vec{u}-\vec{v}\|$

$= \|\overrightarrow{(0,-2,-1,1)} - \overrightarrow{(-3,2,4,4)}\| = \|\overrightarrow{(3,-4,-5,-3)}\|$

$= \sqrt{3^2+(-4)^2+(-5)^2+(-3)^2}$

$= \sqrt{9+16+25+9} = \sqrt{59}$;

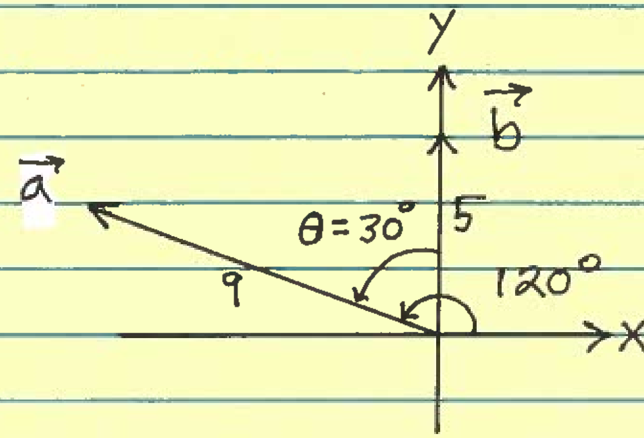
$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \frac{0-4-4+4}{\sqrt{0^2+(-2)^2+(-1)^2+1^2}\sqrt{(-3)^2+2^2+4^2+4^2}}$$

$$= \frac{-4}{\sqrt{6}\sqrt{45}} = \frac{-4}{\sqrt{150}},$$ so θ is obtuse

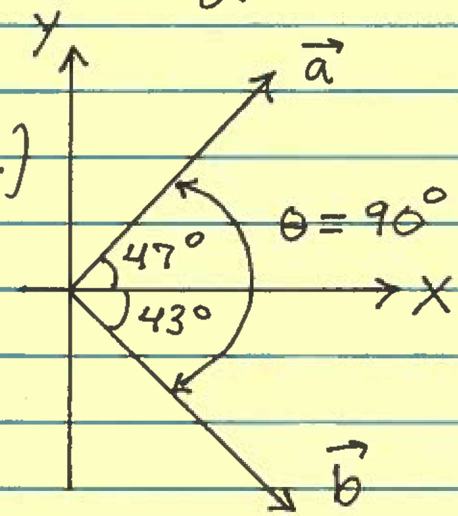
13.) $\vec{a}\cdot\vec{b} = \|\vec{a}\|\,\|\vec{b}\|\cos\theta$

$= (9)(5)\cos 30^\circ$

$= 45\cdot\frac{\sqrt{3}}{2}$



14.)



$\vec{a}\cdot\vec{b} = \|\vec{a}\|\,\|\vec{b}\|\cos\theta$

$= \|\vec{a}\|\,\|\vec{b}\|\cos 90^\circ$

$= \|\vec{a}\|\,\|\vec{b}\|\cdot(0)$

$= 0$

15.) a.) NO b.) YES c.) NO d.) YES
since $\vec{v}\cdot\vec{w}$ is a #, not a vector
since $\vec{u}\cdot\vec{v}$ is a #, not a vector

16.) a.) NO (since $\|\vec{u}\|$ and $\|\vec{v}\|$ are not vectors b.) NO (since $\vec{u}\cdot\vec{v}$ is not a vector c.) YES d.) NO (since k is not a vector)

18.) a) $\vec{u} = \overrightarrow{(4,1,1)}$, $\vec{v} = \overrightarrow{(1,2,3)}$ so

$\|\vec{u}\| = \sqrt{4^2+1^2+1^2} = \sqrt{18} = 3\sqrt{2}$;

$\|\vec{v}\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$;

$\|\vec{u}\| \cdot \|\vec{v}\| = 3\sqrt{2} \cdot \sqrt{2 \cdot 7} = 3\sqrt{2}\sqrt{2}\sqrt{7} = 6\sqrt{7}$;

$|\vec{u} \circ \vec{v}| = |4+2+3| = 9$; then

$|\vec{u} \bullet \vec{v}| = 9 \leq 6\sqrt{7} = \|\vec{u}\| \, \|\vec{v}\|$

b.) $\vec{u} = \overrightarrow{(1,2,1,2,3)}$, $\vec{v} = \overrightarrow{(0,1,1,5,-2)}$ so

$\|\vec{u}\| = \sqrt{1^2+2^2+1^2+2^2+3^2} = \sqrt{19}$;

$\|\vec{v}\| = \sqrt{0^2+1^2+1^2+5^2+(-2)^2} = \sqrt{31}$;

$\|\vec{u}\| \, \|\vec{v}\| = \sqrt{19}\sqrt{31} = \sqrt{589}$;

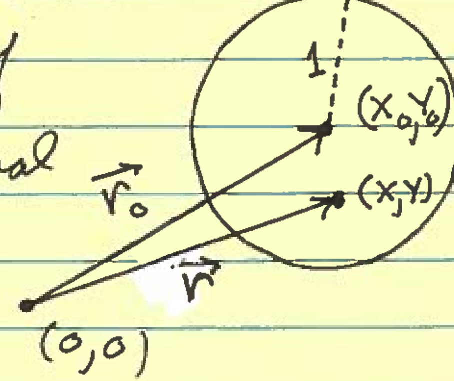
$|\vec{u} \bullet \vec{v}| = |0+2+1+10-6| = 6$; then

$|\vec{u} \bullet \vec{v}| = 6 \leq \sqrt{589} = \|\vec{u}\| \, \|\vec{v}\|$

19.) b.) $\vec{r}_0 = \overrightarrow{(x_0, y_0)}$, $\vec{r} = \overrightarrow{(x, y)}$:

$\|\vec{r} - \vec{r}_0\| \leq 1$ is the set of all vectors with terminal pts. (x,y) on or inside the circle

$(x-x_0)^2 + (y-y_0)^2 = 1^2$



21.) $\vec{v} = \overrightarrow{(v_1, v_2, v_3)}$, $\vec{i} = \overrightarrow{(1,0,0)}$, $\vec{j} = \overrightarrow{(0,1,0)}$, $\vec{k} = \overrightarrow{(0,0,1)}$:

angle α between $\vec{i}$ and $\vec{v}$:

$$\cos\alpha = \frac{\vec{i}\cdot\vec{v}}{\|\vec{i}\|\cdot\|\vec{v}\|} = \frac{v_1}{1\,\|v\|} = \frac{v_1}{\|v\|};$$

angle β between $\vec{j}$ and $\vec{v}$:

$$\cos\beta = \frac{\vec{j}\cdot\vec{v}}{\|\vec{j}\|\,\|\vec{v}\|} = \frac{v_2}{1\,\|\vec{v}\|} = \frac{v_2}{\|\vec{v}\|};$$

angle γ between $\vec{k}$ and $\vec{v}$:

$$\cos\gamma = \frac{\vec{k}\cdot\vec{v}}{\|\vec{k}\|\,\|\vec{v}\|} = \frac{v_3}{1\,\|\vec{v}\|} = \frac{v_3}{\|v\|}$$

22.) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$

$$= \left(\frac{v_1}{\|\vec{v}\|}\right)^2 + \left(\frac{v_2}{\|\vec{v}\|}\right)^2 + \left(\frac{v_3}{\|\vec{v}\|}\right)^2$$

$$= \frac{v_1^2}{\|\vec{v}\|^2} + \frac{v_2^2}{\|\vec{v}\|^2} + \frac{v_3^2}{\|\vec{v}\|^2}$$

$$= \frac{\left(\sqrt{v_1^2 + v_2^2 + v_3^2}\right)^2}{\|\vec{v}\|^2}$$

$$= \frac{\|\vec{v}\|^2}{\|\vec{v}\|^2} = 1.$$

23.) Assume $\vec{v} \neq \vec{0}$ with direction cosines:

$$\cos\alpha_1 = \frac{v_1}{\|\vec{v}\|},\ \cos\beta_1 = \frac{v_2}{\|\vec{v}\|},\ \cos\gamma_1 = \frac{v_3}{\|\vec{v}\|};$$

Assume $\vec{w} \neq \vec{0}$ with direction cosines:

$$\cos\alpha_2 = \frac{w_1}{\|\vec{w}\|},\ \cos\beta_2 = \frac{w_2}{\|\vec{w}\|},\ \cos\gamma_2 = \frac{w_3}{\|\vec{w}\|};\ \text{then}$$

$$\vec{v}\cdot\vec{w} = \overrightarrow{(v_1, v_2, v_3)} \cdot \overrightarrow{(w_1, w_2, w_3)}$$

$$= v_1 w_1 + v_2 w_2 + v_3 w_3$$

$$= \|\vec{v}\|\cos\alpha_1 \cdot \|\vec{w}\|\cos\alpha_2$$

$$+ \|\vec{v}\|\cos\beta_1 \cdot \|\vec{w}\|\cos\beta_2 + \|\vec{v}\|\cos\gamma_1 \cdot \|\vec{w}\|\cos\gamma_2$$

$$= \|\vec{v}\|\|\vec{w}\|(\cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_1\cos\gamma_2)$$

(where $\|\vec{v}\| \neq 0$, $\|\vec{w}\| \neq 0$)

($\Rightarrow$): assume $\vec{v} \perp \vec{w}$, then $\vec{v}\cdot\vec{w} = 0$,

so that $\cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_1\cos\gamma_2 = 0$.

($\Leftarrow$): Assume $\cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_1\cos\gamma_2 = 0$,

then $\vec{v}\cdot\vec{w} = 0$, so that $\vec{v} \perp \vec{w}$.

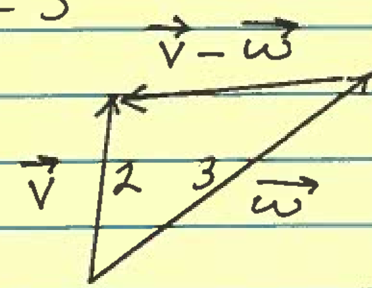
24.) a.) Without loss of generality, assume that the cube has edge length 1. Then (SEE diagram)

$\vec{d} = \overrightarrow{(1,1,1)}$ and $\vec{u} = \overrightarrow{(1,1,0)}$ and

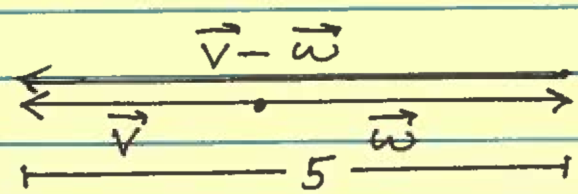
$$\cos\theta = \frac{\vec{d}\cdot\vec{u}}{\|\vec{d}\|\,\|\vec{u}\|} = \frac{1+1+0}{\sqrt{3}\cdot\sqrt{2}} = \frac{2}{\sqrt{6}} \Rightarrow$$

$$\theta = \arccos\left(\frac{2}{\sqrt{6}}\right) \approx 35.3^\circ$$

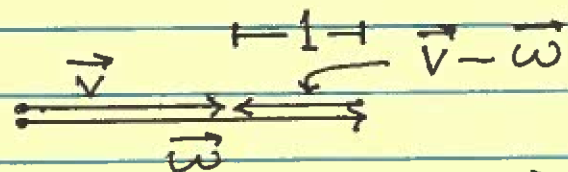
26.) Assume $\|\vec{v}\| = 2$, $\|\vec{w}\| = 3$

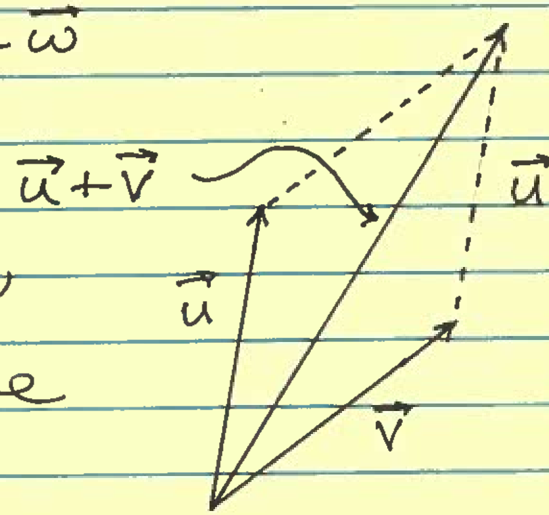


i.) MAX $\|\vec{v}-\vec{w}\|$ is 5:

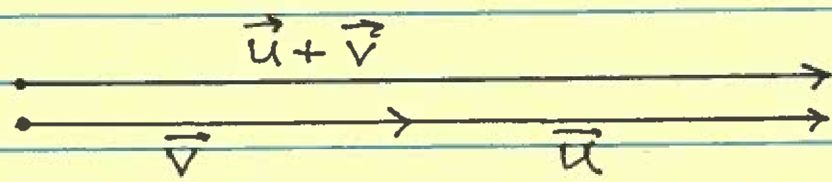


ii.) MIN $\|\vec{v}-\vec{w}\|$ is 1:





27.) If $\|\vec{u}+\vec{v}\| = \|\vec{u}\| + \|\vec{v}\|$, then $\vec{u}$ and $\vec{v}$ point in the same direction:



30.) (SEE problem 27.)

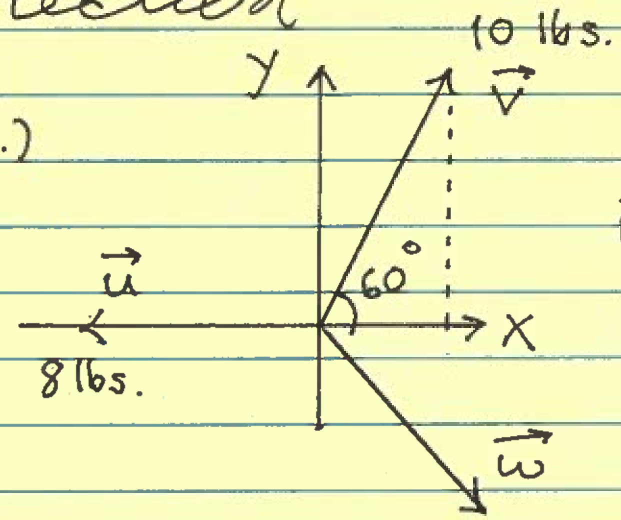
$\|\vec{u}+\vec{v}\| \le \|\vec{u}\|+\|\vec{v}\|$ becomes

$\|\vec{u}+\vec{v}\| = \|\vec{u}\|+\|\vec{v}\|$ if

i.) $\vec{u}=\vec{0}$ or ii.) $\vec{v}=\vec{0}$ or

iii) $\vec{u}$ and $\vec{v}$ point in the same direction

32.) a.)



We want $\vec{u}+\vec{v}+\vec{w}=\vec{0} \Rightarrow$

$\overrightarrow{(-8,0)}+\overrightarrow{(10\cos 60°, 10\sin 60°)}+\vec{w}=\vec{0} \Rightarrow$

$\overrightarrow{(-8,0)}+\overrightarrow{(10(1/2), 10(\sqrt{3}/2))}+\vec{w}=\vec{0} \Rightarrow$

$\overrightarrow{(-8,0)}+\overrightarrow{(5, 5\sqrt{3})}+\vec{w}=\vec{0} \Rightarrow$

$\overrightarrow{(-3, 5\sqrt{3})}+\vec{w}=\vec{0} \Rightarrow$

$\vec{w}=\overrightarrow{(3, -5\sqrt{3})}$, where

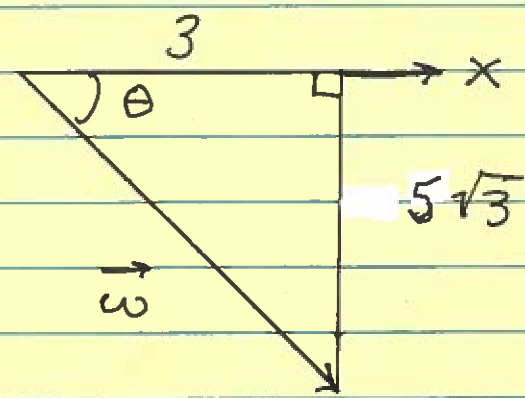
$\|\vec{w}\|=\sqrt{3^2+(-5\sqrt{3})^2}=\sqrt{9+75}=\sqrt{84}$

$= 2\sqrt{21}$ lbs.,

and

$\theta = \arctan\left(\frac{5\sqrt{3}}{3}\right)$

$\approx 70.89^\circ$



TRUE/FALSE

(a) T (b) T (c) F (d) T

(e) T (f) F (g) F (h) F

(i) T (j) T