Section 3.3

1.) a.) $\vec{u} \cdot \vec{v} = \overrightarrow{(6,1,4)} \cdot \overrightarrow{(2,0,-3)} = 12 + 0 - 12 = 0$, so YES, $\vec{u} \perp \vec{v}$

b.) $\vec{u} \cdot \vec{v} = \overrightarrow{(0,0,-1)} \cdot \overrightarrow{(1,1,1)} = 0 + 0 - 1 = -1 \neq 0$, so NO

c.) $\vec{u} \cdot \vec{v} = \overrightarrow{(3,-2,1,3)} \cdot \overrightarrow{(-4,1,-3,7)}$
$= -12 - 2 - 3 + 21 = 4 \neq 0$, so NO

d.) $\vec{u} \cdot \vec{v} = \overrightarrow{(5,-4,0,3)} \cdot \overrightarrow{(-4,1,-3,7)}$

$= -20 - 4 + 0 + 21 = -3 \neq 0$, so NO

2.) b.) $\vec{u} \cdot \vec{v} = \overrightarrow{(1,1,1)} \cdot \overrightarrow{(0,0,0)} = 0 + 0 + 0 = 0$, so YES, $\vec{u} \perp \vec{v}$

c.) $\vec{u} \cdot \vec{v} = \overrightarrow{(1,-5,4)} \cdot \overrightarrow{(3,3,3)} = 3 - 15 + 12 = 0$,

so YES, $\vec{u} \perp \vec{v}$

3.) $P = (-1,3,-2)$, $\vec{n} = \overrightarrow{(-2,1,-1)}$, then

plane is $-2(x-(-1)) + 1 \cdot (y-3) + -1 \cdot (z-(-2)) = 0$

$\Rightarrow -2(x+1) + (y-3) - (z+2) = 0$

4.) $P = (1,1,4)$, $\vec{n} = \overrightarrow{(1,9,8)}$, then plane

is $1 \cdot (x-1) + 9(y-1) + 8(z-4) = 0$

5.) $P = (2,0,0)$, $\vec{n} = \overrightarrow{(0,0,2)}$, then plane is $0(x-2)+0(y-0)+2(z-0)=0 \Rightarrow$
$z=0$

7.) $4x-y+2z=5 \Rightarrow \vec{n_1} = \overrightarrow{(4,-1,2)}$;
$7x-3y+4z=8 \Rightarrow \vec{n_2} = \overrightarrow{(7,-3,4)} \Rightarrow$
NOT parallel
since $\vec{n_1} \neq k\vec{n_2}$

8.) $x-4y-3z-2=0 \Rightarrow \vec{n_1} = \overrightarrow{(1,-4,-3)}$;
$3x-12y-9z-7=0 \Rightarrow \vec{n_2} = \overrightarrow{(3,-12,-9)} \Rightarrow$
PARALLEL since $\vec{n_2} = 3\vec{n_1}$

9.) $2y = 8x-4z+5 \Rightarrow -8x+2y+4z=5$
$\Rightarrow \vec{n_1} = \overrightarrow{(-8,2,4)}$;
$x = \frac{1}{2}z + \frac{1}{4}y \Rightarrow 4x = 2z+y \Rightarrow$
$4x-y-2z=0 \Rightarrow \vec{n_2} = \overrightarrow{(4,-1,-2)} \Rightarrow$
PARALLEL since $\vec{n_1} = -2\vec{n_2}$

11.) $3x-y+z-4=0 \Rightarrow \vec{n_1} = \overrightarrow{(3,-1,1)}$;
$x+2z=-1 \Rightarrow \vec{n_2} = \overrightarrow{(1,0,2)}$, then
$\vec{n_1} \cdot \vec{n_2} = 3+0+2 = 5 \neq 0$, so NOT $\perp$

12.) $x-2y+3z=4 \Rightarrow \vec{n_1}=\overrightarrow{(1,-2,3)}$;

$-2x+5y+4z=-1 \Rightarrow \vec{n_2}=\overrightarrow{(-2,5,4)}$, then

$\vec{n_1}\cdot\vec{n_2}=-2-10+12=0$, so YES $\perp$

13.) a.) $\vec{u}=\overrightarrow{(1,-2)}$, $\vec{a}=\overrightarrow{(-4,-3)}$, then

i.) $proj_{\vec{a}}\vec{u}=\left(\dfrac{\vec{u}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\right)\vec{a}=\left(\dfrac{-4+6}{16+9}\right)\vec{a}$

$=\dfrac{2}{25}\overrightarrow{(-4,-3)}=\overrightarrow{\left(\dfrac{-8}{25},\dfrac{-6}{25}\right)}$;

ii.) $\|proj_{\vec{a}}\vec{u}\|=\dfrac{|\vec{u}\cdot\vec{a}|}{\|\vec{a}\|}=\dfrac{|2|}{5}=\dfrac{2}{5}$

b.) $\vec{u}=\overrightarrow{(3,0,4)}$, $\vec{a}=\overrightarrow{(2,3,3)}$, then

i.) $proj_{\vec{a}}\vec{u}=\left(\dfrac{\vec{u}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\right)\vec{a}=\dfrac{6+0+12}{4+9+9}\vec{a}=\dfrac{18}{22}\vec{a}$

$=\dfrac{9}{11}\overrightarrow{(2,3,3)}=\overrightarrow{\left(\dfrac{18}{11},\dfrac{27}{11},\dfrac{27}{11}\right)}$;

ii.) $\|proj_{\vec{a}}\vec{u}\|=\dfrac{|\vec{u}\cdot\vec{a}|}{\|\vec{a}\|}=\dfrac{18}{\sqrt{22}}$

14.) a.) $\vec{u}=\overrightarrow{(5,6)}$, $\vec{a}=\overrightarrow{(2,-1)}$, then

i.) $proj_{\vec{a}}\vec{u}=\left(\dfrac{\vec{u}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\right)\vec{a}=\dfrac{10-6}{4+1}\vec{a}$

$=\dfrac{4}{5}\overrightarrow{(2,-1)}=\overrightarrow{\left(\dfrac{8}{5},\dfrac{-4}{5}\right)}$;

ii.) $\|\text{proj}_{\vec{a}}\vec{u}\| = \dfrac{|\vec{u}\cdot\vec{a}|}{\|\vec{a}\|} = \dfrac{4}{\sqrt{5}}$

b.) $\vec{u} = \overrightarrow{(3,-2,6)}$, $\vec{a} = \overrightarrow{(1,2,-7)}$, then

i.) $\text{proj}_{\vec{a}}\vec{u} = \left(\dfrac{\vec{u}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\right)\vec{a} = \dfrac{3-4-42}{1+4+49}\vec{a}$

$= \dfrac{-43}{54}\overrightarrow{(1,2,-7)} = \overrightarrow{\left(\dfrac{-43}{54}, \dfrac{-43}{27}, \dfrac{301}{54}\right)}$;

ii.) $\|\text{proj}_{\vec{a}}\vec{u}\| = \dfrac{|\vec{u}\cdot\vec{a}|}{\|\vec{a}\|} = \dfrac{|-43|}{\sqrt{54}} = \dfrac{43}{3\sqrt{6}}$

15.) $\vec{u} = \overrightarrow{(6,2)}$, $\vec{a} = \overrightarrow{(3,-9)}$, then

i.) $\text{proj}_{\vec{a}}\vec{u} = \left(\dfrac{\vec{u}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\right)\vec{a} = \dfrac{18-18}{9+81}\vec{a} = 0\,\vec{a} = \overrightarrow{(0,0)}$

ii.) $\vec{u} - \text{proj}_{\vec{a}}\vec{u} = \overrightarrow{(6,2)} - \overrightarrow{(0,0)} = \overrightarrow{(6,2)}$

16.) $\vec{u} = \overrightarrow{(-1,-2)}$, $\vec{a} = \overrightarrow{(-2,3)}$, then

i.) $\text{proj}_{\vec{a}}\vec{u} = \left(\dfrac{\vec{u}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\right)\vec{a} = \dfrac{2-6}{4+9}\vec{a}$

$= \dfrac{-4}{13}\overrightarrow{(-2,3)} = \overrightarrow{\left(\dfrac{8}{13}, \dfrac{-12}{13}\right)}$

ii.) $\vec{u} - \text{proj}_{\vec{a}}\vec{u} = \overrightarrow{(-1,-2)} - \overrightarrow{\left(\dfrac{8}{13}, \dfrac{-12}{13}\right)}$

$= \overrightarrow{\left(\dfrac{-13}{13}, \dfrac{-26}{13}\right)} + \overrightarrow{\left(\dfrac{-8}{13}, \dfrac{12}{13}\right)} = \overrightarrow{\left(\dfrac{-21}{13}, \dfrac{-14}{13}\right)}$

17.) $\vec{u} = \overrightarrow{(3,1,-7)}$, $\vec{a} = \overrightarrow{(1,0,5)}$, then

i.) $\text{proj}_{\vec{a}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\right)\vec{a} = \frac{3+0-35}{1+0+25}\vec{a}$

$= \frac{-32}{26}\vec{a} = \frac{-16}{13}\overrightarrow{(1,0,5)} = \overrightarrow{\left(\frac{-16}{13}, 0, \frac{-80}{13}\right)}$

ii.) $\vec{u} - \text{proj}_{\vec{a}}\vec{u} = \overrightarrow{(3,1,-7)} - \overrightarrow{\left(\frac{-16}{13}, 0, \frac{-80}{13}\right)}$

$= \overrightarrow{\left(\frac{39}{13}, \frac{13}{13}, \frac{-91}{13}\right)} + \overrightarrow{\left(\frac{16}{13}, 0, \frac{80}{13}\right)} = \overrightarrow{\left(\frac{55}{13}, 1, \frac{-11}{13}\right)}$

20.) $\vec{u} = \overrightarrow{(5,0,-3,7)}$, $\vec{a} = \overrightarrow{(2,1,-1,-1)}$, then

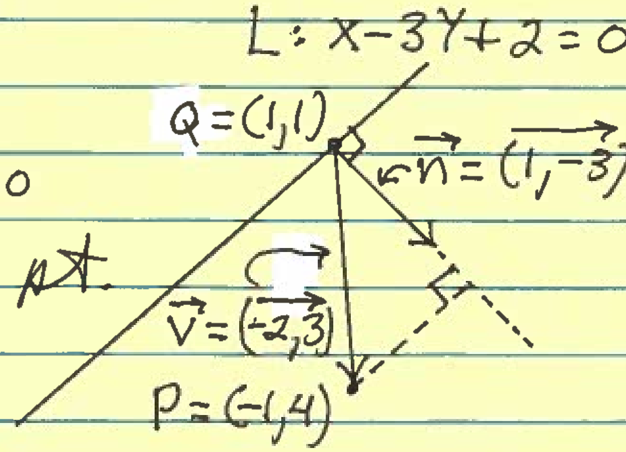
i.) $\text{proj}_{\vec{a}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\right)\vec{a} = \frac{10+0+3-7}{4+1+1+1}\vec{a}$

$= \frac{6}{7}\overrightarrow{(2,1,-1,-1)} = \overrightarrow{\left(\frac{12}{7}, \frac{6}{7}, \frac{-6}{7}, \frac{-6}{7}\right)}$

ii.) $\vec{u} - \text{proj}_{\vec{a}}\vec{u} = \overrightarrow{(5,0,-3,7)} - \overrightarrow{\left(\frac{12}{7}, \frac{6}{7}, \frac{-6}{7}, \frac{-6}{7}\right)}$

$= \overrightarrow{\left(\frac{35}{7}, 0, \frac{-21}{7}, \frac{49}{7}\right)} + \overrightarrow{\left(\frac{-12}{7}, \frac{-6}{7}, \frac{6}{7}, \frac{6}{7}\right)}$

$= \overrightarrow{\left(\frac{23}{7}, \frac{-6}{7}, \frac{-15}{7}, \frac{55}{7}\right)}$



22.) $P = (-1,4)$, $x - 3y + 2 = 0$

$\Rightarrow \vec{n} = \overrightarrow{(1,-3)}$, and pt.

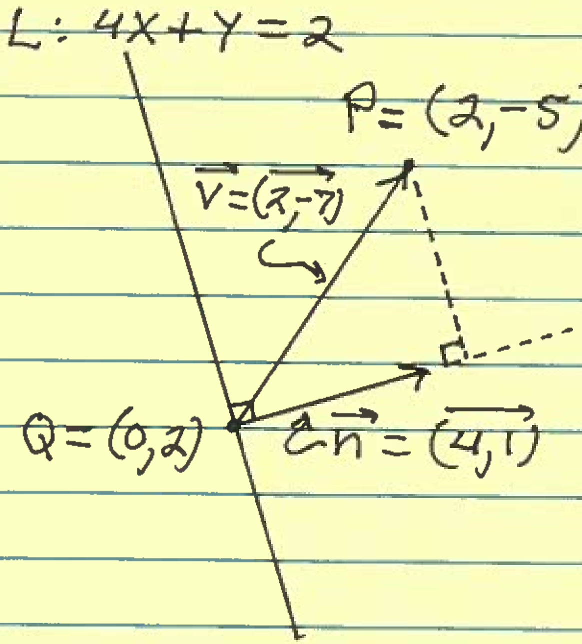
$Q = (1,1)$ lies on line:

Distance from pt. P to line L is

$$\|\text{proj}_{\vec{n}}\vec{v}\| = \frac{|\vec{v}\cdot\vec{n}|}{\|\vec{n}\|} = \frac{|-2-9|}{\sqrt{10}} = \frac{11}{\sqrt{10}}$$

23.) $P = (2,-5)$,

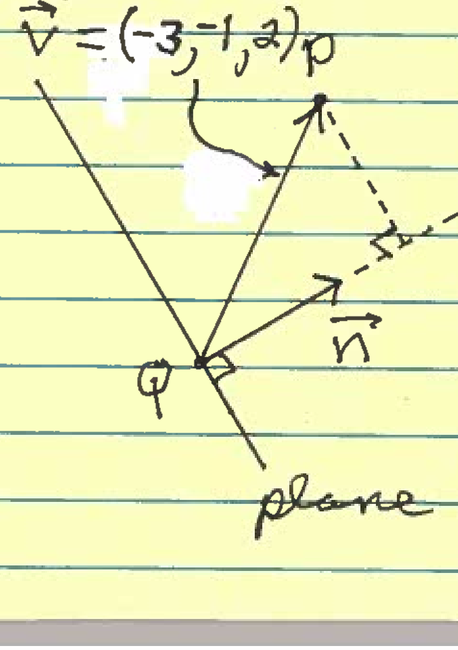
$4X + Y = 2 \Rightarrow$
$\vec{n} = \overrightarrow{(4,1)}$, and pt.
$Q = (0,2)$ lies
on line :



Distance from
pt. P to line L is

$$\|\text{proj}_{\vec{n}}\vec{v}\| = \frac{|\vec{v}\cdot\vec{n}|}{\|\vec{n}\|} = \frac{|8-7|}{\sqrt{17}} = \frac{1}{\sqrt{17}}$$

26.) $P = (-1,-1,2)$, plane $2X + 5Y - 6Z = 4$
$\Rightarrow \vec{n} = \overrightarrow{(2,5,-6)}$, and pt.
$Q = (2,0,0)$ lies on
the plane :
Distance from pt.
P to plane is

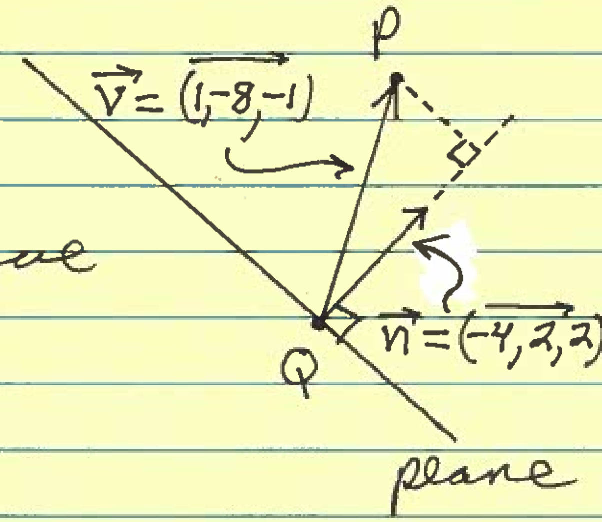


$$\|\text{proj}_{\vec{n}}\vec{v}\| = \frac{|\vec{v}\cdot n|}{\|\vec{n}\|}$$

$$= \frac{|-6-5-12|}{\sqrt{4+25+36}} = \frac{23}{\sqrt{65}}$$

27.) Point $P=(1,-2,-1)$ lies on plane $2x-y-z=5$; find the distance from pt. P to the other plane $-4x+2y+2z=12$:

pt. $Q=(0,6,0)$ is on the plane, so distance from P to the other plane is



$$\|\text{proj}_{\vec{n}}\vec{v}\| = \frac{|\vec{v}\cdot\vec{n}|}{\|\vec{n}\|}$$

$$= \frac{|-4-16-2|}{\sqrt{16+4+4}} = \frac{22}{\sqrt{24}} = \frac{22}{2\sqrt{6}} = \frac{11}{\sqrt{6}}$$

29.) Let $\vec{w}=\overrightarrow{(a,b,c)}$; we want

$\vec{w}\perp\vec{u} \Rightarrow \vec{w}\cdot\vec{u}=0 \Rightarrow$

$\overrightarrow{(a,b,c)}\cdot\overrightarrow{(1,0,1)} = a+0+c=0 \Rightarrow \boxed{a+c=0}$;

$\vec{w}\perp\vec{v} \Rightarrow \vec{w}\cdot\vec{v}=0 \Rightarrow$

$\overrightarrow{(a,b,c)}\cdot\overrightarrow{(0,1,1)} = 0+b+c=0 \Rightarrow \boxed{b+c=0}$;

then $c=-a \Rightarrow$ (sub) $b+(-a)=0 \rightarrow b=a$, so let $c=1$, then $a=-1$, $b=-1$ and

$\vec{w} = \overrightarrow{(-1,-1,1)}$ with $\|\vec{w}\| = \sqrt{3}$; thus

$\frac{1}{\sqrt{3}}\overrightarrow{(-1,-1,1)} = \overrightarrow{\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$ is unit vector

$\perp$ to both $\vec{u}$ and $\vec{v}$

30.) a.) $\vec{v} \cdot \vec{w} = \overrightarrow{(a,b)} \cdot \overrightarrow{(-b,a)} = -ab + ab = 0$,

so $\vec{v} \perp \vec{w}$

b.) $\vec{v} = \overrightarrow{(2,-3)}$, so $\overrightarrow{(3,2)}$ and $\overrightarrow{(-3,-2)}$

are $\perp \vec{v}$

c.) $\vec{v} = \overrightarrow{(-3,4)}$, so $\overrightarrow{(-4,-3)}$ and $\overrightarrow{(4,3)}$

are $\perp \vec{v}$

31.) $A=(3,0,2)$, $B=(4,3,0)$, $C=(8,1,-1)$;

length $\overrightarrow{AB} = \sqrt{(4-3)^2+(3-0)^2+(0-2)^2}$

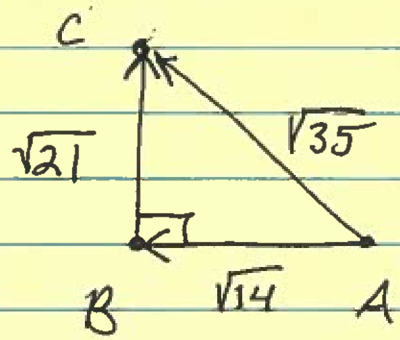
$= \sqrt{1+9+4} = \sqrt{14}$;

length $\overrightarrow{AC} = \sqrt{(8-3)^2+(1-0)^2+(-1-2)^2}$

$= \sqrt{25+1+9} = \sqrt{35}$;

length $\overrightarrow{BC} = \sqrt{(8-4)^2+(1-3)^2+(-1-0)^2}$

$= \sqrt{16+4+1} = \sqrt{21}$, then

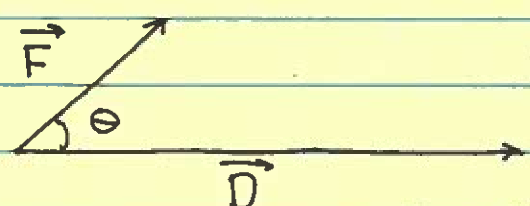


$(\sqrt{14})^2 + (\sqrt{21})^2 = (\sqrt{35})^2$, i.e.,
$14 + 21 = 35$, so
that $\triangle ABC$ is a
right triangle

33.) assume $\vec{v} \perp \vec{w}_1$ and $\vec{v} \perp \vec{w}_2$.
Show $\vec{v} \perp (k_1\vec{w}_1 + k_2\vec{w}_2)$:

$$\vec{v} \cdot (k_1\vec{w}_1 + k_2\vec{w}_2) = \vec{v} \cdot (k_1\vec{w}_1) + \vec{v} \cdot (k_2\vec{w}_2)$$
$$= k_1(\vec{v} \cdot w_1) + k_2(\vec{v} \cdot \vec{w}_2)$$
$$= k_1(0) + k_2(0) \quad (\text{since } \vec{v} \perp \vec{w}_1, \vec{v} \perp \vec{w}_2)$$
$$= 0 + 0 = 0, \text{ so } \vec{v} \perp (k_1\vec{w}_1 + k_2\vec{w}_2)$$

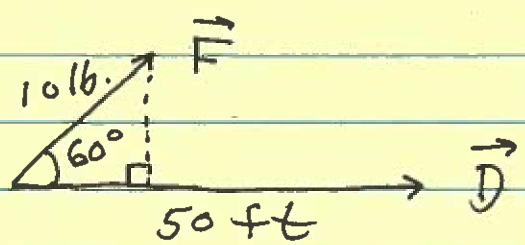
34.) YES; if $\vec{u} = \vec{a} \neq \vec{0}$, ~~then~~
$proj_{\vec{a}}\vec{u} = proj_{\vec{u}}\vec{a}$.



$\vec{F}$: force vector
$\vec{D}$: distance vector

then Work $= \vec{F} \cdot \vec{D}$

36.)



Then

$$\text{Work} = \vec{F} \cdot \vec{D}$$

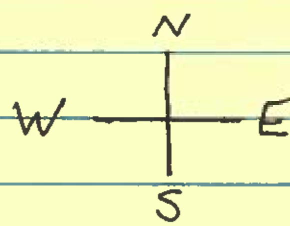
$$= \overrightarrow{(10\cos 60°, 10\sin 60°)} \cdot \overrightarrow{(50, 0)}$$

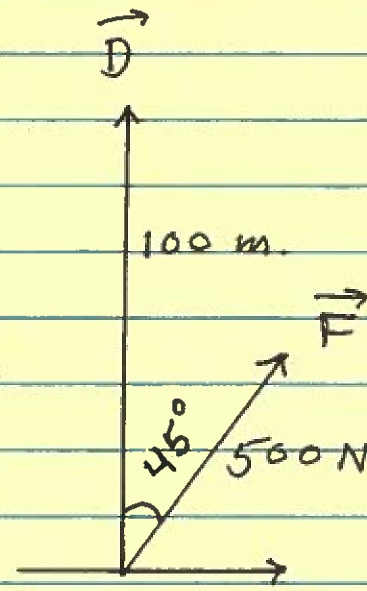
$$= \overrightarrow{(10(1/2), 10(\sqrt{3}/2))} \cdot \overrightarrow{(50, 0)}$$

$$= \overrightarrow{(5, 5\sqrt{3})} \cdot \overrightarrow{(50, 0)}$$

$$= 250 \text{ ft-lbs.}$$

37.) Then





$$\text{Work} = \vec{F} \cdot \vec{D}$$

$$= \overrightarrow{(500\cos 45°, 500\cos 45°)} \cdot \overrightarrow{(0, 100)}$$

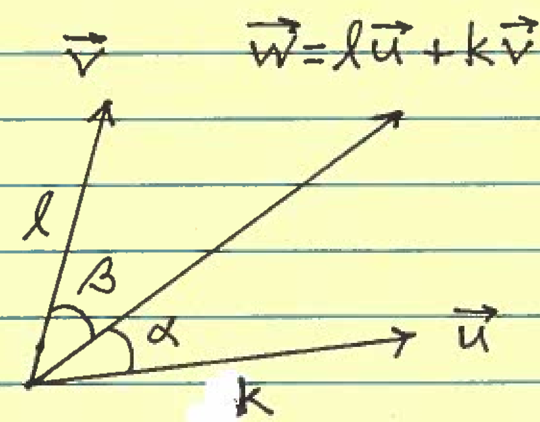
$$= \overrightarrow{\left(500 \cdot \frac{1}{\sqrt{2}}, 500 \cdot \frac{1}{\sqrt{2}}\right)} \cdot \overrightarrow{(0, 100)}$$

$$= 0 + 500 \cdot \frac{1}{\sqrt{2}} \cdot 100$$

$$= \frac{50{,}000}{\sqrt{2}} \text{ N.m.}$$

38.) Let $\vec{u}$ and $\vec{v}$ be vectors with $\|\vec{u}\| = k$ and $\|\vec{v}\| = l$. Show that vector $\vec{w} = l\vec{u} + k\vec{v}$ bisects the angle between $\vec{u}$ and $\vec{v}$:

Show $\cos\alpha = \cos\beta$, so that $\alpha = \beta$.



i.) First, note that

$\vec{u} \cdot \vec{w} = \vec{u} \cdot (l\vec{u} + k\vec{v})$

$= l(\vec{u} \cdot \vec{u}) + k(\vec{u} \cdot \vec{v})$

$= \|\vec{v}\| \cdot \|\vec{u}\|^2 + \|\vec{u}\| \cdot (\vec{u} \cdot \vec{v})$

$= \|\vec{u}\| \{\|\vec{u}\| \|\vec{v}\| + (\vec{u} \cdot \vec{v})\}$;

ii.) Second, note that

$\vec{v} \cdot \vec{w} = \vec{v} \cdot (l\vec{u} + k\vec{v})$

$= l(\vec{v} \cdot \vec{u}) + k(\vec{v} \cdot \vec{v})$

$= \|\vec{v}\| (\vec{u} \cdot \vec{v}) + \|\vec{u}\| \cdot \|\vec{v}\|^2$

$= \|\vec{v}\| \{\|\vec{u}\| \|\vec{v}\| + (\vec{u} \cdot \vec{v})\}$;

Now find $\cos\alpha$ and $\cos\beta$:

$$\cos\alpha = \frac{\vec{u} \cdot \vec{w}}{\|\vec{u}\| \|\vec{w}\|} = \frac{\cancel{\|\vec{u}\|} \{\|\vec{u}\| \|\vec{v}\| + (\vec{u} \cdot \vec{v})\}}{\cancel{\|\vec{u}\|} \cdot \|\vec{w}\|}$$

$$= \frac{\|\vec{u}\|\|\vec{v}\| + (\vec{u} \cdot \vec{v})}{\|\vec{w}\|}, \text{ and}$$

$$\cos\beta = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\|\|\vec{w}\|} = \frac{\cancel{\|\vec{v}\|}\{\|\vec{u}\|\|\vec{v}\| + (\vec{u} \cdot \vec{v})\}}{\cancel{\|\vec{v}\|}\,\|\vec{w}\|}$$

$$= \frac{\|\vec{u}\|\|\vec{v}\| + (\vec{u} \cdot \vec{v})}{\|\vec{w}\|}, \text{ so that}$$

$$\cos\alpha = \cos\beta \Rightarrow \alpha = \beta$$

TRUE/FALSE

(a) T (b) T (c) T (d) T

(e) T (f) F (g) F