Section 2.1

1.) $a_{11} = 1$, $M_{11} = \begin{vmatrix} 7 & -1 \\ 1 & 4 \end{vmatrix} = 28 - (-1) = 29$

and $C_{11} = (-1)^2 M_{11} = 29$;

$a_{12} = -2$, $M_{12} = \begin{vmatrix} 6 & -1 \\ -3 & 4 \end{vmatrix} = 24 - 3 = 21$

and $C_{12} = (-1)^3 21 = -21$;

$a_{13} = 3$, $M_{13} = \begin{vmatrix} 6 & 7 \\ -3 & 1 \end{vmatrix} = 6 - (-21) = 27$

and $C_{13} = (-1)^4 27 = 27$;

$a_{21} = 6$, $M_{21} = \begin{vmatrix} -2 & 3 \\ 1 & 4 \end{vmatrix} = -8 - 3 = -11$

and $C_{21} = (-1)^3 (-11) = 11$;

$a_{22} = 7$, $M_{22} = \begin{vmatrix} 1 & 3 \\ -3 & 4 \end{vmatrix} = 4 - (-9) = 13$

and $C_{22} = (-1)^4 (13) = 13$;

$a_{23} = -1$, $M_{23} = \begin{vmatrix} 1 & -2 \\ -3 & 1 \end{vmatrix} = 1 - 6 = -5$

and $C_{23} = (-1)^5 (-5) = 5$

$a_{31} = -3$, $M_{31} = \begin{vmatrix} -2 & 3 \\ 7 & -1 \end{vmatrix} = 2 - 21 = -19$

and $C_{31} = (-1)^4 (-19) = -19$

$a_{32} = 1$, $M_{32} = \begin{vmatrix} 1 & 3 \\ 6 & -1 \end{vmatrix} = -1 - 18 = -19$

and $C_{32} = (-1)^5 (-19) = 19$

$a_{33} = 4$, $M_{33} = \begin{vmatrix} 1 & -2 \\ 6 & 7 \end{vmatrix} = 7 - (-12) = 19$

and $C_{33} = (-1)^6 19 = 19$

3.) a.) $a_{13} = 1$ so

$$M_{13} = \begin{vmatrix} 0 & 0 & 3 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{vmatrix} = 0 \begin{vmatrix} 1 & 14 \\ 1 & 2 \end{vmatrix} - 0 \begin{vmatrix} 4 & 14 \\ 4 & 2 \end{vmatrix} + 3 \begin{vmatrix} 4 & 1 \\ 4 & 1 \end{vmatrix}$$

$$= 0 - 0 + 3(4-4) = 0,$$

$C_{13} = (-1)^4 (0) = 0$

c.) $a_{22} = 0$ so

$$M_{22} = \begin{vmatrix} 4 & 1 & 6 \\ 4 & 0 & 14 \\ 4 & 3 & 2 \end{vmatrix} = 4 \begin{vmatrix} 0 & 14 \\ 3 & 2 \end{vmatrix} - 1 \begin{vmatrix} 4 & 14 \\ 4 & 2 \end{vmatrix} + 6 \begin{vmatrix} 4 & 0 \\ 4 & 3 \end{vmatrix}$$

$$= 4(0-42) - (8-56) + 6(12-0)$$

$$= -168 + 48 + 72 = -48,$$

$C_{22} = (-1)^4 (-48) = -48$

4.) c.) $a_{41} = 3$ so

$$M_{41} = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ -2 & 1 & 0 \end{vmatrix} = 3 \begin{vmatrix} 0 & 3 \\ 1 & 0 \end{vmatrix} - (-1) \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} + 1 \begin{vmatrix} 2 & 0 \\ -2 & 1 \end{vmatrix}$$

$$= 3(0-3) + (2-(-6)) + (2-0)$$

$$= -9 + 8 + 2 = 1,$$

$C_{41} = (-1)^5 (1) = -1$

5.) $\det \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix} = 12 - (-10) = 22 \neq 0$, so

$$A^{-1} = \frac{1}{22}\begin{bmatrix} 4 & -5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2/11 & -5/22 \\ 1/11 & 3/22 \end{bmatrix}$$

6.) $\det \begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix} = 8 - 8 = 0$, so A is not invertible

8.) $\det \begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix} = \sqrt{6} - 4\sqrt{6} = -3\sqrt{6} \neq 0$, so

$$A^{-1} = \frac{1}{-3\sqrt{6}}\begin{bmatrix} \sqrt{3} & -\sqrt{6} \\ -4 & \sqrt{2} \end{bmatrix} = \begin{bmatrix} -1/3\sqrt{2} & 1/3 \\ 4/3\sqrt{6} & -1/3\sqrt{3} \end{bmatrix}$$

9.) $\begin{vmatrix} a-3 & 5 \\ -3 & a-2 \end{vmatrix} = (a-3)(a-2) - (-15)$

$= a^2 - 5a + 6 + 15 = a^2 - 5a + 21$

10.) $\begin{vmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{vmatrix} = \begin{bmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{bmatrix} \begin{matrix} -2 & 7 \\ 5 & 1 \\ 3 & 8 \end{matrix}$

$= (-8) + (-42) + (240) - (18) - (32) - (140) = 0$

12.) $\begin{vmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{vmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{bmatrix} \begin{matrix} -1 & 1 \\ 3 & 0 \\ 1 & 7 \end{matrix}$

$= (0) + (-5) + (42) - (0) - (35) - (6) = -4$

15.) $\begin{vmatrix} \lambda-2 & 1 \\ -5 & \lambda+4 \end{vmatrix} = (\lambda-2)(\lambda+4) - (-5)$

$= \lambda^2 + 2\lambda - 8 + 5$

$= \lambda^2 + 2\lambda - 3 = (\lambda-1)(\lambda+3) = 0 \Rightarrow \lambda = 1 \text{ or } \lambda = -3$

16.) $\begin{vmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{vmatrix} = (\lambda-4)\begin{vmatrix} \lambda & 2 \\ 3 & \lambda-1 \end{vmatrix} - 0\begin{vmatrix} 0 & 2 \\ 0 & \lambda-1 \end{vmatrix} + 0\begin{vmatrix} 0 & \lambda \\ 0 & 3 \end{vmatrix}$

$= (\lambda-4)[\lambda(\lambda-1)-6] = (\lambda-4)(\lambda^2-\lambda-6)$

$= (\lambda-4)(\lambda-3)(\lambda+2) = 0 \Rightarrow \lambda = 4, \lambda = 3, \text{ or } \lambda = -2$

17.) $\begin{vmatrix} \lambda-1 & 0 \\ 2 & \lambda+1 \end{vmatrix} = (\lambda-1)(\lambda+1) - 0 = 0 \Rightarrow$

$\lambda = 1 \text{ or } \lambda = -1$

19.) c.) $\begin{vmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{vmatrix} = 2(-1)^3\begin{vmatrix} 0 & 0 \\ 9 & -4 \end{vmatrix} + (-1)(-1)^4\begin{vmatrix} 3 & 0 \\ 1 & -4 \end{vmatrix}$

$+ 5(-1)^5\begin{vmatrix} 3 & 0 \\ 1 & 9 \end{vmatrix}$

$= -2(0-0) - (-12-0) - 5(27-0)$

$= 12 - 135 = -123$

d.) $\begin{vmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{vmatrix} = 0(-1)^4\begin{vmatrix} 2 & -1 \\ 1 & 9 \end{vmatrix} + 5(-1)^5\begin{vmatrix} 3 & 0 \\ 1 & 9 \end{vmatrix}$

$+ (-4)(-1)^6\begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix}$

$= 0 - 5(27-0) - 4(-3-0)$

$= -135 + 12 = -123$

21.) $\begin{vmatrix} -3 & 0 & 7 \\ 2 & 5 & 1 \\ -1 & 0 & 5 \end{vmatrix} = (0)(-1)^3 \begin{vmatrix} 2 & 1 \\ -1 & 5 \end{vmatrix} + 5(-1)^4 \begin{vmatrix} -3 & 7 \\ -1 & 5 \end{vmatrix}$

$+ 0(-1)^5 \begin{vmatrix} -3 & 7 \\ 2 & 1 \end{vmatrix}$

$= 5(-15-(-7)) = 5(-8) = -40$

23.) $\begin{vmatrix} 1 & k & k^2 \\ 1 & k & k^2 \\ 1 & k & k^2 \end{vmatrix} = (1)\begin{vmatrix} k & k^2 \\ k & k^2 \end{vmatrix} - (1)\begin{vmatrix} k & k^2 \\ k & k^2 \end{vmatrix} + (1)\begin{vmatrix} k & k^2 \\ k & k^2 \end{vmatrix}$

$= (k^3-k^3)-(k^3-k^3)+(k^3-k^3) = 0$

28.) $\begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{vmatrix} = (2)(2)(2) = 8$

29.) $\begin{vmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 4 & 3 & 0 \\ 1 & 2 & 3 & 8 \end{vmatrix} = (0)(2)(3)(8) = 0$

33.) a.) $\begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix} = \sin^2\theta - (-\cos^2\theta)$

$= \sin^2\theta + \cos^2\theta = 1$

b.) $\begin{vmatrix} \sin\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ \sin\theta-\cos\theta & \sin\theta+\cos\theta & 1 \end{vmatrix}$

$= (0)\begin{vmatrix} x & x \\ x & x \end{vmatrix} - (0)\begin{vmatrix} x & x \\ x & x \end{vmatrix} + (1)(-1)^6 \begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix}$

$= \sin^2\theta + \cos^2\theta = 1$

34.) Assume $A=\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ and $B=\begin{bmatrix} d & e \\ 0 & f \end{bmatrix}$. Note that

$$AB=\begin{bmatrix} ad & ae+bf \\ 0 & cf \end{bmatrix},$$

$$BA=\begin{bmatrix} ad & bd+ce \\ 0 & cf \end{bmatrix}, \text{ and}$$

$$\begin{vmatrix} b & a-c \\ e & d-f \end{vmatrix} = b(d-f)-e(a-c)$$
$$= bd-bf-ae+ce$$
$$= bd+ce-ae-bf.$$

Show that $AB=BA$ iff $bd+ce-ae-bf=0$:

($\Rightarrow$) assume $AB=BA$, then

$bd+ce=ae+bf \Rightarrow bd+ce-ae-bf=0$

($\Leftarrow$) assume $bd+ce-ae-bf=0$,

then $bd+ce=ae+bf \Rightarrow AB=BA$

35.) $$d_1=\begin{vmatrix} a & b & c \\ d & 1 & f \\ g & 0 & 1 \end{vmatrix} = a(1-0)-bX+cY,$$

$$d_2=\begin{vmatrix} a+\lambda & b & c \\ d & 1 & f \\ g & 0 & 1 \end{vmatrix} = (a+\lambda)(1-0)-bX+cY$$
$$= d_1+\lambda$$

38.) Six zeroes: $\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}$, where a, b, c are $\neq 0$

TRUE / FALSE

(a) F (b) F (c) T (d) T

(e) T (f) T (g) F (h) F

(i) F (j) T