Section 4.1

1.) Let $V = \mathbb{R}^2$, $\vec{u} = (u_1, u_2)$, $\vec{v} = (v_1, v_2)$,
where $\vec{u} + \vec{v} = (u_1 + v_1, u_2 + v_2)$
and
$$k\vec{u} = (0, ku_2)$$

a.) $\vec{u} + \vec{v} = (-1, 2) + (3, 4) = (-1+3, 2+4)$
$= (2, 6)$;
$k\vec{u} = 3(-1, 2) = (-3, 6)$

b.) Clearly, $\vec{u} + \vec{v} = (u_1 + v_1, u_2 + v_2) \in \mathbb{R}^2$;
clearly, $k\vec{u} = (0, ku_2) \in \mathbb{R}^2$

c.) axioms 1–5

d.) axiom 7:

$k(\vec{u} + \vec{v}) = k((u_1, u_2) + (v_1, v_2))$

$= k(u_1 + v_1, u_2 + v_2)$ (by definition)

$= (0, k(u_2 + v_2))$ (by definition)

$= (0, ku_2 + kv_2)$ (distributive property)

$= (0, ku_2) + (0, kv_2)$ (by definition)

$= k(u_1, u_2) + k(v_1, v_2)$ (by definition)

$= k\vec{u} + k\vec{v}$

axiom 8:

$(k+m)\vec{u} = (k+m)(u_1, u_2)$

$= (0, (k+m)u_2)$ (by definition)

$= (0, ku_2 + mu_2)$ (distributive property)

$= (0, ku_2) + (0, mu_2)$ (by definition)

$= k(u_1, u_2) + m(u_1, u_2)$

$= k\vec{u} + m\vec{u}$

axiom 9:

$k(m\vec{u}) = k(m(u_1, u_2))$

$= k(0, mu_2)$ (by definition)

$= (0, k(mu_2))$ (by definition)

$= (0, (km)u_2)$ (associative property)

$= (km)(u_1, u_2)$ (by definition)

e.) Let $\vec{u} = (3,3)$, then

$1(3,3) = (0, (1)(3))$ (by definition)

$= (0,3) \neq (3,3) = \vec{u}$, so

axiom 10 FAILS

2.) Let $V = \mathbb{R}^2$, $\vec{u} = (u_1, u_2)$, $\vec{v} = (v_1, v_2)$, where $\vec{u} + \vec{v} = (u_1 + v_1 + 1, u_2 + v_2 + 1)$ and

$$k\vec{u} = (ku_1, ku_2)$$

a.) $\vec{u} + \vec{v} = (0, 4) + (1, {}^-3)$

$= (0 + 1 + 1, 4 + {}^-3 + 1) = (2, 2)$;

$k\vec{u} = 2(0, 4) = (2 \cdot 0, 2 \cdot 4) = (0, 8)$

b.) $(u_1, u_2) + (0, 0) = (u_1 + 0 + 1, u_2 + 0 + 1)$

$= (u_1 + 1, u_2 + 1) \neq (u_1, u_2)$, so

$(0, 0) \neq \vec{0}$

c.) $(u_1, u_2) + ({}^-1, {}^-1) = (u_1 + {}^-1 + 1, u_2 + {}^-1 + 1)$

$= (u_1, u_2)$

and

$({}^-1, {}^-1) + (u_1, u_2) = ({}^-1 + u_1 + 1, {}^-1 + u_2 + 1) = (u_1, u_2)$,

so $({}^-1, {}^-1) = \vec{0}$

d.) axiom 5: For $\vec{u} = (u_1, u_2)$ let

$-\vec{u} = ({}^-u_1 + {}^-2, {}^-u_2 + {}^-2)$, then

$\vec{u} + {}^-\vec{u} = (u_1, u_2) + ({}^-u_1 + {}^-2, {}^-u_2 + {}^-2)$

$= (u_1 + {}^-u_1 + {}^-2 + 1, u_2 + {}^-u_2 + {}^-2 + 1) = ({}^-1, {}^-1) = \vec{0}$

$-\vec{u} + \vec{u} = ({}^-u_1 + {}^-2, {}^-u_2 + {}^-2) + (u_1, u_2)$

$= (-u_1 + -2 + u_1 + 1, -u_2 + -2 + u_2 + 1) = (-1,-1) = \vec{0}$

e.) axiom 7: Let $k=2$, $\vec{u}=(0,0)$, $\vec{v}=(3,3)$

then $k(\vec{u}+\vec{v}) = 2((0,0)+(3,3))$

$= 2(0+3+1, 0+3+1) = 2(4,4) = (8,8)$; but

$k\vec{u} + k\vec{v} = 2(0,0) + 2(3,3)$

$= (0,0) + (6,6) = (0+6+1, 0+6+1) = (7,7)$,

so axiom 7 FAILS

axiom 8: Let $k=1$, $m=1$, $\vec{u}=(3,3)$

then $(k+m)\vec{u} = (1+1)(3,3) = 2(3,3) = (6,6)$;

but

$k\vec{u} + m\vec{u} = 1(3,3) + 1(3,3)$

$= (3,3) + (3,3) = (3+3+1, 3+3+1) = (7,7)$,

so axiom 8 FAILS

4.) $V = \{(x,0) \mid (x,0) \in \mathbb{R}^2\}$, $\vec{u} = (u_1, 0)$,

$\vec{v} = (v_1, 0)$, where

$\vec{u} + \vec{v} = (u_1 + v_1, 0)$, $k\vec{u} = (ku_1, 0)$:

axiom 1: $\vec{u}+\vec{v} = (u_1+v_1, 0) \in V$ (TRUE)

axiom 2: $\vec{u}+\vec{v} = (u_1+v_1, 0) = (v_1+u_1, 0)$

$= (v_1, 0) + (u_1, 0) = \vec{v} + \vec{u}$ (TRUE)

<u>axiom 3</u>: $\vec{u} + (\vec{v} + \vec{w}) = (u_1, 0) + ((v_1, 0) + (w_1, 0))$

$= (u_1, 0) + (v_1 + w_1, 0) = (u_1 + (v_1 + w_1), 0)$

$= ((u_1 + v_1) + w_1, 0) = (u_1 + v_1, 0) + (w_1, 0)$

$= ((u_1, 0) + (v_1, 0)) + (w_1, 0)$

$= (\vec{u} + \vec{v}) + \vec{w}$ (TRUE)

<u>axiom 4</u>: Let $\vec{0} = (0, 0)$, then

$\vec{u} + \vec{0} = (u_1, 0) + (0, 0) = (u_1 + 0, 0) = (u_1, 0) = \vec{u}$,

$\vec{0} + \vec{u} = (0, 0) + (u_1, 0) = (0 + u_1, 0) = (u_1, 0) = \vec{u}$

(TRUE)

<u>axiom 5</u>: For $\vec{u} = (u_1, 0)$, let

$-\vec{u} = (-u_1, 0)$, then

$\vec{u} + -\vec{u} = (u_1, 0) + (-u_1, 0) = (u_1 + -u_1, 0) = (0, 0) = \vec{0}$,

$-\vec{u} + \vec{u} = (-u_1, 0) + (u_1, 0) = (-u_1 + u_1, 0) = (0, 0) = \vec{0}$

(TRUE)

<u>axiom 6</u>: $k\vec{u} = k(u_1, 0) = (ku_1, 0) \in V$ (TRUE)

<u>axiom 7</u>: $k(\vec{u} + \vec{v}) = k((u_1, 0) + (v_1, 0))$

$= k(u_1 + v_1, 0) = (k(u_1 + v_1), 0)$

$= (ku_1 + kv_1, 0) = (ku_1, 0) + (kv_1, 0)$

$= k(u_1,0) + k(v_1,0) = k\vec{u} + k\vec{v}$ (TRUE)

<u>axiom 8</u>: $(k+m)\vec{u} = (k+m)(u_1,0)$

$= ((k+m)u_1, 0) = (ku_1 + mu_1, 0)$

$= (ku_1, 0) + (mu_1, 0) = k(u_1,0) + m(u_1,0)$

$= k\vec{u} + m\vec{u}$ (TRUE)

<u>axiom 9</u>: $k(m\vec{u}) = k(m(u_1,0))$

$= k(mu_1, 0) = (k(mu_1), 0) = ((km)u_1, 0)$

$= (km)(u_1,0) = (km)\vec{u}$ (TRUE)

<u>axiom 10</u>: $1\vec{u} = 1(u_1,0) = (1u_1, 0)$

$= (u_1, 0) = \vec{u}$ (TRUE)

So V is a vector space

5.) $V = \{(x,y) \mid (x,y) \in \mathbb{R}$ and $x \geq 0\}$,

$\vec{u} = (u_1, u_2)$, $\vec{v} = (v_1, v_2)$, where

$\vec{u} + \vec{v} = (u_1 + v_1, u_2 + v_2)$ and

$k\vec{u} = k(u_1, u_2) = (ku_1, ku_2)$:

<u>Axioms</u> 1, 2, 3, 4 ($\vec{0} = (0,0)$), 7, 8, 9, 10

are TRUE ;

axiom 5 : If $\vec{u} = (u_1, u_2)$ and $-\vec{u} = (-u_1, -u_2)$, then

$$\vec{u} + -\vec{u} = (u_1, u_2) + (-u_1, -u_2)$$
$$= (u_1 + -u_2, u_2 + -u_2) = (0,0) = \vec{0},$$ but

$(-u_1, -u_2) \notin V$, since $-u_1 < 0$.

axiom 6 : If $\vec{u} = (u_1, u_2)$ and $k < 0$, $u_1 > 0$, then $k\vec{u} = k(u_1, u_2) = (ku_1, ku_2) \notin V$ since $ku_1 < 0$;

So V is NOT a vector space

6.) $V = \{(x, x, \ldots, x) \mid (x, x, \ldots, x) \in \mathbb{R}^n\}$,

$\vec{u} = (u, u, \ldots, u)$, $\vec{v} = (v, v, \ldots, v)$, where

$\vec{u} + \vec{v} = (u+v, u+v, \ldots, u+v)$ and

$k\vec{u} = (ku, ku, \ldots, ku)$:

axiom 1 : $\vec{u} + \vec{v} = (u+v, u+v, \ldots, u+v) \in V$ (TRUE)

axiom 2 : $\vec{u} + \vec{v} = (u+v, u+v, \ldots, u+v)$

$= (v+u, v+u, \ldots, v+u) = (v, v, \ldots, v) + (u, u, \ldots, u)$

$= \vec{v} + \vec{u}$ (TRUE)

axiom 3 : $\vec{u} + (\vec{v} + \vec{w})$

$= (u,u,\dots,u) + ((v,v,\dots,v) + (w,w,\dots,w))$
$= (u,u,\dots,u) + (v+w, v+w, \dots, v+w)$
$= (u+(v+w), u+(v+w), \dots, u+(v+w))$
$= ((u+v)+w, (u+v)+w, \dots, (u+v)+w)$
$= (u+v, u+v, \dots, u+v) + (w,w,\dots,w)$
$= ((u,u,\dots,u)+(v,v,\dots,v)) + (w,w,\dots,w)$
$= (\vec{u}+\vec{v}) + \vec{w}$ (TRUE)

<u>axiom 4</u>: Let $\vec{0} = (0,0,\dots,0)$, then
$\vec{u}+\vec{0} = (u,u,\dots,u)+(0,0,\dots,0)$
$= (u+0, u+0, \dots, u+0) = (u,u,\dots,u) = \vec{u}$,
$\vec{0}+\vec{u} = (0,0,\dots,0) + (u,u,\dots,u)$
$= (0+u, 0+u, \dots, 0+u) = (u,u,\dots,u) = \vec{u}$ (TRUE)

<u>axiom 5</u>: If $\vec{u} = (u,u,\dots,u)$ ~~and~~ $-\vec{u} = (-u,-u,\dots,-u)$
then $\vec{u} + -\vec{u} = (u,u,\dots,u) + (-u,-u,\dots,-u)$
$= (u+-u, u+-u, \dots, u+-u) = (0,0,\dots,0) = \vec{0}$,
$-\vec{u}+\vec{u} = (-u,-u,\dots,-u)+(u,u,\dots,u)$
$= (-u+u, -u+u, \dots, -u+u) = (0,0,\dots,0) = \vec{0}$ (TRUE)

<u>axiom 6</u>: $k\vec{u} = k(u,u,\dots,u) = (ku, ku, \dots, ku) \in V$ (TRUE)

<u>axiom 7</u>: $k(\vec{u}+\vec{v}) = k((u,u,\dots,u)+(v,v,\dots,v))$

$= k(u+v, u+v, \ldots, u+v) = (k(u+v), k(u+v), \ldots, k(u+v))$

$= (ku+kv, ku+kv, \ldots, ku+kv)$

$= (ku, ku, \ldots, ku) + (kv, kv, \ldots, kv)$

$= k(u, u, \ldots, u) + k(v, v, \ldots, v) = k\vec{u} + k\vec{v}$ (TRUE)

<u>Axiom 8</u>: $(k+m)\vec{u} = (k+m)(u, u, \ldots, u)$

$= ((k+m)u, (k+m)u, \ldots, (k+m)u)$

$= (ku+mu, ku+mu, \ldots, ku+mu)$

$= (ku, ku, \ldots, ku) + (mu, mu, \ldots, mu)$

$= k(u, u, \ldots, u) + m(u, u, \ldots, u)$

$= k\vec{u} + m\vec{u}$ (TRUE)

<u>Axiom 9</u>: $k(m\vec{u}) = k(m(u, u, \ldots, u))$

$= k(mu, mu, \ldots, mu) = (k(mu), k(mu), \ldots, k(mu))$

$= ((km)u, (km)u, \ldots, (km)u)$

$= (km)(u, u, \ldots, u) = (km)\vec{u}$ (TRUE)

<u>Axiom 10</u>: $1\vec{u} = 1(u, u, \ldots, u)$

$= (1u, 1u, \ldots, 1u) = (u, u, \ldots, u) = \vec{u}$ (TRUE),

so V is a vector space

7.) $V = \{(x,y,z) \mid (x,y,z) \in \mathbb{R}^3\}$,

$\vec{u} = (x_1, y_1, z_1)$, $\vec{v} = (x_2, y_2, z_2)$; where

$\vec{u} + \vec{v} = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$ and

$k\vec{u} = (k^2 x_1, k^2 x_2, k^2 x_3)$:

<u>axioms</u> 1, 2, 3, 4, and 5 are TRUE;

<u>axiom 6</u> : $k\vec{u} = k(x_1, y_1, z_1) = (k^2 x_1, k^2 y_1, k^2 z_1) \in V$ (TRUE)

<u>axiom 7</u> : $k(\vec{u}+\vec{v}) = k((x_1,y_1,z_1) + (x_2,y_2,z_2))$

$= k(x_1 + x_2, y_1 + y_2, z_1 + z_2)$

$= (k^2(x_1+x_2), k^2(y_1+y_2), k^2(z_1+z_2))$

$= (k^2 x_1 + k^2 x_2, k^2 y_1 + k^2 y_2, k^2 z_1 + k^2 z_2)$

$= (k^2 x_1, k^2 y_1, k^2 z_1) + (k^2 x_2, k^2 y_2, k^2 z_2)$

$= k(x_1, y_1, z_1) + k(x_2, y_2, z_2) = k\vec{u} + k\vec{v}$ (TRUE)

<u>axiom 8</u> : $(k+m)\vec{u} = (k+m)(x_1, y_1, z_1)$

$= ((k+m)^2 x_1, (k+m)^2 y_1, (k+m)^2 z_1)$

$= ((k^2+2km+m^2)x_1, (k^2+2km+m^2)y_1, (k^2+2km+m^2)z_1)$

$= (k^2 x_1 + m^2 x_1 + 2km\,x_1, k^2 y_1 + 2km\,y_1 + m^2 y_1,$

$k^2 z_1 + 2km\,z_1 + m^2 z_1)$

$= (k^2x_1, k^2y_1, k^2z_1) + (m^2x_1, m^2y_1, m^2z_1)$
$\quad + (2kmx_1, 2kmy_1, 2kmz_1)$

$= k(x_1,y_1,z_1) + m(x_1,y_1,z_1) + (2kmx_1, 2kmy_1, 2kmz_1)$

$= k\vec{u} + m\vec{u} + (2kmx_1, 2kmy_1, 2kmz_1)$

$\neq k\vec{u} + m\vec{u}$ (NOT TRUE)

axiom 9: $k(mu) = k(m(x_1,y_1,z_1))$

$= k(m^2x_1, m^2y_1, m^2z_1)$

$= (k^2(m^2x_1), k^2(m^2y_1), k^2(m^2z_1))$

$= ((k^2m^2)x_1, (k^2m^2)y_1, (k^2m^2)z_1)$

$= ((km)^2x_1, (km)^2y_1, (km)^2z_1)$

$= (km)(x_1,y_1,z_1) = (km)\vec{u}$ (TRUE)

axiom 10: $1\vec{u} = 1(x_1,y_1,z_1)$

$= (1^2x_1, 1^2y_1, 1^2z_1) = (1x_1, 1y_1, 1z_1) = (x_1,y_1,z_1)$

$= \vec{u}$ (TRUE)

So V is NOT a vector space

8.) $V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is invertible,} \right.$
i.e., $\left. \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} \neq 0 \right\}$,

$\vec{u} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\vec{v} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, where

$\vec{u} + \vec{v} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$, $k\vec{u} = \begin{bmatrix} ka, & kb \\ kc, & kd \end{bmatrix}$:

<u>axiom 1</u>: If $\vec{u} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$,
then $\vec{u} \in V$ and $\vec{v} \in V$ since
$\det \vec{u} = 1 \neq 0$ and $\det \vec{v} = 1 \neq 0$, but
$\vec{u} + \vec{v} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \notin V$,
so NOT TRUE

<u>axiom 4</u>: $\vec{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is the only
possibility, but $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \notin V$, since $\det \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$,
so NOT TRUE

<u>axiom 6</u>: If $k = 0$ and $\vec{u} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$,
then $\vec{u} \in V$ since $\det \vec{u} = 6 \neq 0$, but
$k\vec{u} = (0) \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \notin V$, since
$\det \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$, so NOT TRUE

<u>axioms</u> 2, 3, 5, 7, 8, 9, 10 are TRUE,
so V is NOT a vector space

12.) $V = \{a+bx \mid a,b \in \mathbb{R}\}$, $\vec{u} = a+bx$,

$\vec{v} = c+dx$, ~~where~~

$\vec{u}+\vec{v} = (a+c)+(b+d)x$ and

$k\vec{u} = ka+(kb)x$:

<u>Axiom 1</u>: $\vec{u}+\vec{v} = (a+c)+(b+d)x \in V$

since $a+c \in \mathbb{R}$, $b+d \in \mathbb{R}$ (TRUE)

<u>Axiom 2</u>: $\vec{u}+\vec{v} = (a+c)+(b+d)x$

$= (c+a)+(d+b)x = (c+dx)+(a+bx)$

$= \vec{v}+\vec{u}$ (TRUE)

<u>Axiom 3</u>: $\vec{u}+(\vec{v}+\vec{w}) = (a+bx)+((c+dx)+(e+fx))$

$= (a+bx)+((c+e)+(d+f)x)$

$= (a+(c+e))+(b+(d+f))x$

$= ((a+c)+e)+((b+d)+f)x$

$= ((a+c)+(b+d)x)+(e+fx)$

$= ((a+bx)+(c+dx))+(e+fx)$

$= (\vec{u}+\vec{v})+\vec{w}$ (TRUE)

<u>Axiom 4</u>: Let $\vec{0} = 0+(0)x = 0$ then

$\vec{u}+\vec{0} = (a+bx)+(0+(0)x) = (a+0)+(b+0)x$

$= a+bx = \vec{u}$ and

$\vec{0}+\vec{u} = (0+(0)x)+(a+bx) = (0+a)+(0+b)x$

$= a+bx = \vec{u}$ (TRUE)

<u>axiom 5</u>: If $\vec{u} = a + bx$, let $-\vec{u} = -a + (-b)x$

then $\vec{u} + -\vec{u} = (a+bx) + (-a + (-b)x)$

$= (a + -a) + (b + -b)x = 0 + (0)x = \vec{0}$ and

$-\vec{u} + \vec{u} = (-a + (-b)x) + (a + bx)$

$= (-a + a) + (-b + b)x = 0 + (0)x = \vec{0}$ (TRUE)

<u>axiom 6</u>: $k\vec{u} = k(a+bx) = ka + (kb)x \in V$ (TRUE)

<u>axiom 7</u>: $k(\vec{u} + \vec{v}) = k((a+bx) + (c+dx))$

$= k((a+c) + (b+d)x)$

$= k(a+c) + k(b+d)x$

$= (ka + kc) + (kb + kd)x$

$= (ka + (kb)x) + (kc + (kd)x)$

$= k(a+bx) + k(c+dx) = k\vec{u} + k\vec{v}$ (TRUE)

<u>axiom 8</u>: $(k+m)\vec{u} = (k+m)(a+bx)$

$= (k+m)a + (k+m)bx$

$= (ka + ma) + (kb + mb)x$

$= (ka + (kb)x) + (ma + (mb)x)$

$= k(a+bx) + m(a+bx) = k\vec{u} + m\vec{u}$ (TRUE)

<u>axiom 9</u>: $k(m\vec{u}) = k(m(a+bx))$

$= k(ma + (mb)x) = k(ma) + k(mb)x$

$= (km)a + (km)bx = (km)(a+bx)$

$= (km)\vec{u}$ (TRUE)

axiom 10: $1\vec{u} = 1(a+bx)$
$= 1\cdot a + (1\cdot b)x = a + bx = \vec{u}$ (TRUE)
so V is a vector space

17.) ($\Rightarrow$): assume that
$V = \{(x,y) \mid (x,y) \in \mathbb{R}^2 \text{ and } y = mx + b\}$
with standard vector addition and scalar multiplication is a vector space. Show that $b=0$ (line passes through the origin.):
By axiom 4 $\vec{0} = (0,0) \in V \Rightarrow$
$0 = m(0) + b \Rightarrow b = 0$

($\Leftarrow$): Let $V = \{(x,y) \mid (x,y) \in \mathbb{R}^2 \text{ and } y = mx\}$
(line passes through the origin.)
with standard vector addition and scalar multiplication. Show that V is a vector space:
axiom 1: $\vec{u} = (x_1, y_1)$, $\vec{v} = (x_2, y_2) \Rightarrow$
$\vec{u} + \vec{v} = (x_1 + x_2, y_1 + y_2)$ and
$y_1 + y_2 = mx_1 + mx_2 = m(x_1 + x_2) \Rightarrow$
$\vec{u} + \vec{v} \in V$ (TRUE)

<u>axiom 2</u> : $\vec{u}+\vec{v} = (x_1, y_1) + (x_2, y_2)$

$= (x_1 + x_2, y_1 + y_2) = (x_2 + x_1, y_2 + y_1)$

$= (x_2, y_2) + (x_1, y_1) = \vec{v} + \vec{u}$ (TRUE)

<u>axiom 3</u> : $\vec{u} + (\vec{v} + \vec{w}) = (x_1, y_1) + ((x_2, y_2) + (x_3, y_3))$

$= (x_1, y_1) + (x_2 + x_3, y_2 + y_3)$

$= (x_1 + (x_2 + x_3), y_1 + (y_2 + y_3))$

$= ((x_1 + x_2) + x_3, (y_1 + y_2) + y_3)$

$= (x_1 + x_2, y_1 + y_2) + (x_3, y_3)$

$= ((x_1, y_1) + (x_2, y_2)) + (x_3, y_3)$

$= (\vec{u} + \vec{v}) + \vec{w}$ (TRUE)

<u>axiom 4</u> : Let $\vec{0} = (0,0)$ then

$\vec{u} + \vec{0} = (x, y) + (0, 0) = (x+0, y+0) = (x, y) = \vec{u}$,

$\vec{0} + \vec{u} = (0,0) + (x, y) = (0+x, 0+y) = (x, y) = \vec{u}$

(TRUE)

<u>axiom 5</u> : If $\vec{u} = (x, y)$, then $-\vec{u} = (-x, -y)$

since $\vec{u} + -\vec{u} = (x, y) + (-x, -y) = (x + -x, y + -y)$

$= (0, 0) = \vec{0}$ and

$-\vec{u} + \vec{u} = (-x, -y) + (x, y) = (-x + x, -y + y)$

$= (0, 0) = \vec{0}$

Axiom 6: If $\vec{u} = (x, y) \in V$ then $y = mx$ and $k\vec{u} = k(x,y) = (kx, ky) \in V$ since

$ky = k(mx) = m(kx)$ (TRUE)

Axiom 7: $k(\vec{u}+\vec{v}) = k((x_1, y_1) + (x_2, y_2))$

$= k(x_1+x_2, y_1+y_2) = (k(x_1+x_2), k(y_1+y_2))$

$= (kx_1 + kx_2, ky_1 + ky_2) = (kx_1, ky_1) + (kx_2, ky_2)$

$= k(x_1, y_1) + k(x_2, y_2) = k\vec{u} + k\vec{v}$ (TRUE)

Axiom 8: $(k+m)\vec{u} = (k+m)(x, y)$

$= ((k+m)x, (k+m)y) = (kx + mx, ky + my)$

$= (kx, ky) + (mx, my) = k(x,y) + m(x,y)$

$= k\vec{u} + m\vec{u}$ (TRUE)

Axiom 9: $k(m\vec{u}) = k(m(x,y))$

$= k(mx, my) = (k(mx), k(my))$

$= ((km)x, (km)y) = (km)(x,y) = (km)\vec{u}$ (TRUE)

Axiom 10: $1\vec{u} = 1(x, y)$

$= (1x, 1y) = (x, y) = \vec{u}$ (TRUE)

so V is a vector space.

28.) Assume that $k\vec{u}=\vec{0}$. Show that $k=0$ or $\vec{u}=\vec{0}$:

<u>Case 1</u>: If $k=0$, then

$k\vec{u}=0\vec{u}=\vec{0}$ (by Theorem 4.1.1 (a))

<u>Case 2</u>: If $k\neq 0$, then

$k\vec{u}=\vec{0} \Rightarrow \frac{1}{k}(k\vec{u})=\frac{1}{k}\vec{0}$

$\Rightarrow (\frac{1}{k}k)\vec{u}=\vec{0}$ (by axiom 9 and Theorem 4.1.1 (b))

$\Rightarrow (1)\vec{u}=\vec{0}$ (property of numbers)

$\Rightarrow \vec{u}=\vec{0}$ (by axiom 10)

TRUE/FALSE

(a) T (b) F (c) F (d) F

(e) T (f) F