Section 1.2

3.) a.) $\left[\begin{array}{ccc|c} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 10 & 13 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -37 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{array}\right] \rightarrow x = -37,\ y = -8,\ z = 5$

d.) $\left[\begin{array}{ccc|c} 1 & -3 & 7 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \rightarrow 0 = 1$ so

NO SOLUTION

b.) $\left[\begin{array}{cccc|c} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right] \sim \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & -13 & -10 \\ 0 & 1 & 0 & -13 & -5 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right] \end{array} \rightarrow$

$x_3 + x_4 = 2$ so let $\boxed{x_4 = t}$ any # $\rightarrow$

$\boxed{x_3 = 2 - t}$; $x_2 - 13x_4 = -5 \rightarrow$

$\boxed{x_2 = 13t - 5}$; $x_1 - 13x_4 = -10 \rightarrow$

$\boxed{x_1 = 13t - 10}$

4.) a.) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{array}\right] \rightarrow x = -3,\ y = 0,\ z = 7$

b.) $\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 8 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 1 & -5 \end{array}\right] \end{array} \rightarrow x_3 + x_4 = -5$

so let $\boxed{x_4 = t}$ any # $\rightarrow$

$\boxed{x_3 = -t-5}$; $x_2 + 3x_4 = 2 \longrightarrow$

$\boxed{x_2 = 2-3t}$; $x_1 - 7x_4 = 8 \longrightarrow$

$\boxed{x_1 = 7t+8}$

d.) $\left[\begin{array}{ccc|c} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \to 0=1 \to$ NO SOLUTION

5.) $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -104 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \longrightarrow x = 3,\ y = 1,\ z = 2$

6.) $\left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 3/7 & -1/7 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow x_2 + \frac{4}{7}x_3 = \frac{1}{7} \text{ so}$$

let $\boxed{x_3 = t}$ any # $\rightarrow$ $\boxed{x_2 = \frac{1}{7} - \frac{4}{7}t}$;

$$x_1 + \frac{3}{7}x_3 = \frac{-1}{7} \rightarrow \boxed{x_1 = \frac{-1}{7} - \frac{3}{7}t}$$

7.) $$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right]$$

$$\sim\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow Y - 2Z = 0 \text{ so}$$

let $\boxed{Z = t}$ any #

$\rightarrow$ $\boxed{Y = 2t}$; $X - W = -1$ so let

$\boxed{W = S}$ any # $\rightarrow$ $\boxed{X = S - 1}$

16.) $$\left[\begin{array}{ccc|c} 2 & -1 & -3 & 0 \\ -1 & 2 & -3 & 0 \\ 1 & 1 & 4 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & -3 & -11 & 0 \\ 0 & 3 & 1 & 0 \\ 1 & 1 & 4 & 0 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c} 0 & 0 & -10 & 0 \\ 0 & 1 & 1/3 & 0 \\ 1 & 0 & 11/3 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right] \rightarrow$$

$X = 0, \; Y = 0, \; Z = 0$

17.) $\left[\begin{array}{cccc|c} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 3 & 1 & 1 & 1 & 0 \\ -1 & -3 & -1 & -3 & 0 \end{array}\right]$

$\sim \left[\begin{array}{cccc|c} 0 & -8 & -2 & -8 & 0 \\ 1 & 3 & 1 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 0 & 1 & 1/4 & 1 & 0 \\ 1 & 0 & 1/4 & 0 & 0 \end{array}\right]$

$\rightarrow x_2 + \frac{1}{4}x_3 + x_4 = 0$ so let $\boxed{x_4 = t}$ any #, $\boxed{x_3 = s} \rightarrow \boxed{x_2 = -\frac{1}{4}s - t}$;

$x_1 + \frac{1}{4}x_3 = 0 \longrightarrow \boxed{x_1 = -\frac{1}{4}s}$

19.) $\left[\begin{array}{cccc|c} 0 & 2 & 2 & 4 & 0 \\ 1 & 0 & -1 & -3 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 0 & 2 & 2 & 4 & 0 \\ 1 & 0 & -1 & -3 & 0 \\ 0 & 3 & 3 & 7 & 0 \\ 0 & 1 & 1 & -8 & 0 \end{array}\right]$

$\sim \left[\begin{array}{cccc|c} 0 & 0 & 0 & 20 & 0 \\ 1 & 0 & -1 & -3 & 0 \\ 0 & 0 & 0 & 31 & 0 \\ 0 & 1 & 1 & -8 & 0 \end{array}\right] \sim \begin{array}{c} \begin{array}{ccccc} w & x & y & z & \end{array} \\ \left[\begin{array}{cccc|c} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{array}\right] \end{array} \rightarrow$

$\boxed{z = 0}$; $w - y = 0$ so let $\boxed{y = t}$ any #

$\rightarrow \boxed{w = t}$; $x + y = 0 \longrightarrow \boxed{x = -t}$

20.) $\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 1 & 4 & 2 & 0 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 2 & -4 & 1 & 1 & 0 \\ 1 & -2 & -1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 0 & -10 & 1 & -1 & 0 \\ 0 & -5 & -1 & 0 & 0 \end{array}\right]$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -6 & 4 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 2 & -3 & 0 \\ 0 & 0 & 21 & -11 & 0 \\ 0 & 0 & 9 & -5 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & -6 & 4 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 41/2 & 0 \\ 0 & 0 & 0 & \frac{17}{2} & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow x_1 = 0,\ x_2 = 0,\ x_3 = 0,\ x_4 = 0$$

23.) a.) consistent : unique solution

b.) consistent : infinite # of solutions

c.) no solution

d.) inconclusive : for example ;

if $\left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 0 & * & 0 \\ 0 & 0 & 1 & * \end{array}\right] = \left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & * \end{array}\right]$ then infinite # of solutions ;

if $\left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 0 & * & 0 \\ 0 & 0 & 1 & * \end{array}\right] = \left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right] \sim$

$$\left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right] \rightarrow 0 = 1 \text{ so}$$

NO SOLUTION

26.) $$\begin{array}{c} \begin{array}{cccc} x & y & z & = \end{array} \\ \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & a^2-a+1 & a+1 \end{array}\right] \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 0 & 0 & a^2-a & a-1 \end{array}\right]$$

<u>case 1</u>: If $a^2 - a = 0 \Rightarrow a(a-1) = 0 \Rightarrow$
$a = 0$ OR $a = 1$;
if $\boxed{a=0}$, then the last row of the augmented matrix is $\left[\begin{array}{ccc|c} 0 & 0 & 0 & -1 \end{array}\right]$ so $\boxed{\text{NO Solution}}$;

if $\boxed{a=1}$, then the matrix becomes

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & -1/6 & 1/2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & = \end{array} \\ \sim \left[\begin{array}{ccc|c} 1 & 0 & 2/3 & 1 \\ 0 & 1 & -1/6 & 1/2 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \Rightarrow \begin{cases} x + \frac{2}{3} z = 1 \\ y - \frac{1}{6} z = \frac{1}{2}, \end{cases}$$

so let $\boxed{z = t \text{ any } \#}$ $\Rightarrow$

$\boxed{x = 1 - \frac{2}{3}t}$, $\boxed{y = \frac{1}{2} + \frac{1}{6}t}$ so

<u>infinite # of solutions</u>.

<u>case 2</u>: If $a^2 - a \neq 0$, then last row is $[0 \;\; 0 \;\; a^2-a \,|\, a-1]$

$\sim [0 \;\; 0 \;\; 1 \,|\, \frac{a-1}{a(a-1)} = \frac{1}{a}]$; this will lead to a unique solution.

27.) $\left[\begin{array}{ccc|c} 1 & 3 & -1 & a \\ 1 & 1 & 2 & b \\ 0 & 2 & -3 & c \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & a \\ 0 & -2 & 3 & b-a \\ 0 & 2 & -3 & c \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & a \\ 0 & -2 & 3 & b-a \\ 0 & 0 & 0 & b-a+c \end{array}\right] \to$ for system to be consistent we need

$b - a + c = 0$

29.) $\left[\begin{array}{cc|c} 2 & 1 & a \\ 3 & 6 & b \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & 1 & a \\ 1 & 5 & b-a \end{array}\right]$

$\sim \left[\begin{array}{cc|c} 0 & -9 & -2(b-a)+a \\ 1 & 5 & b-a \end{array}\right]$

$\sim \left[\begin{array}{cc|c} 0 & -9 & 3a-2b \\ 1 & 5 & b-a \end{array}\right]$

$\sim \left[\begin{array}{cc|c} 0 & 1 & -\frac{1}{3}a+\frac{2}{9}b \\ 1 & 5 & b-a \end{array}\right]$

$\sim \left[\begin{array}{cc|c} 0 & 1 & -\frac{1}{3}a+\frac{2}{9}b \\ 1 & 0 & -5(-\frac{1}{3}a+\frac{2}{9}b)+b-a \end{array}\right]$

$\sim \left[\begin{array}{cc|c} 0 & 1 & -\frac{1}{3}a+\frac{2}{9}b \\ 1 & 0 & \frac{2}{3}a-\frac{1}{9}b \end{array}\right] \longrightarrow$

$x = \frac{2}{3}a - \frac{1}{9}b, \; y = -\frac{1}{3}a + \frac{2}{9}b$

30.) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 2 & 0 & 2 & b \\ 0 & 3 & 3 & c \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & -2 & 0 & b-2a \\ 0 & 3 & 3 & c \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & 1 & 0 & -\frac{1}{2}b+a \\ 0 & 3 & 3 & c \end{array}\right]$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & -(-\frac{1}{2}b+a)+a \\ 0 & 1 & 0 & -\frac{1}{2}b+a \\ 0 & 0 & 3 & -3(-\frac{1}{2}b+a)+c \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & \frac{1}{2}b \\ 0 & 1 & 0 & a-\frac{1}{2}b \\ 0 & 0 & 1 & \frac{1}{2}b-a+\frac{1}{3}c \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -(\frac{1}{2}b-a+\frac{1}{3}c)+\frac{1}{2}b \\ 0 & 1 & 0 & a-\frac{1}{2}b \\ 0 & 0 & 1 & \frac{1}{2}b-a+\frac{1}{3}c \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & a-\frac{1}{3}c \\ 0 & 1 & 0 & a-\frac{1}{2}b \\ 0 & 0 & 1 & \frac{1}{2}b-a+\frac{1}{3}c \end{array}\right] \longrightarrow$$

$$x_1 = a-\frac{1}{3}c, \quad x_2 = a-\frac{1}{2}b, \quad x_3 = \frac{1}{2}b-a+\frac{1}{3}c$$

33.) $$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 5 & 3 & 0 \\ -1 & -5 & 5 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & -3 & 8 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 9 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right] \sim \begin{array}{c} \begin{array}{ccc} \sin\alpha & \cos\beta & \tan\gamma \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \end{array} \longrightarrow$$

$$\sin\alpha = 0 \rightarrow \alpha = 0, \pi, 2\pi \ ;$$
$$\cos\beta = 0 \rightarrow \beta = \pi/2, \ 3\pi/2 \ ;$$

$\tan\gamma = 0 \to \gamma = 0, \pi, 2\pi$ so solutions (α, β, γ) are

$(0, \pi/2, 0)$, $(0, \pi/2, \pi)$, $(0, \frac{\pi}{2}, 2\pi)$,
$(0, \frac{3\pi}{2}, 0)$, $(0, 3\pi/2, \pi)$, $(0, 3\pi/2, 2\pi)$,
$(\pi, \pi/2, 0)$, $(\pi, \pi/2, \pi)$, $(\pi, \frac{\pi}{2}, 2\pi)$,
$(\pi, \frac{3\pi}{2}, 0)$, $(\pi, \frac{3\pi}{2}, \pi)$, $(\pi, 3\pi/2, 2\pi)$,
$(2\pi, \pi/2, 0)$, $(2\pi, \pi/2, \pi)$, $(2\pi, \pi/2, 2\pi)$,
$(2\pi, 3\pi/2, 0)$, $(2\pi, \frac{3\pi}{2}, \pi)$, $(2\pi, \frac{3\pi}{2}, 2\pi)$

36.) $\left[\begin{array}{ccc|c} 1 & 2 & -4 & -5 \\ 2 & 3 & 8 & 8 \\ -1 & 9 & 10 & -11 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -4 & -5 \\ 0 & -1 & 16 & 18 \\ 0 & 11 & 6 & -16 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 0 & 28 & 31 \\ 0 & 1 & -16 & -18 \\ 0 & 0 & 182 & 182 \end{array}\right] \sim \begin{array}{c} \begin{array}{ccc} 1/x & 1/y & 1/z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right] \end{array} \longrightarrow$

$\frac{1}{x} = 3, \ \frac{1}{y} = -2, \ \frac{1}{z} = 1 \longrightarrow$

$x = 1/3, \ y = -1/2, \ z = 1$

37.) Plug the four points $(0, 10)$, $(1, 7)$, $(3, -11)$, $(4, -14)$ into the given equation:

$$\begin{cases} \boxed{d=10} \\ a+b+c+10=7 \\ 27a+9b+3c+10=-11 \\ 64a+16b+4c+10=-14 \end{cases} \longrightarrow$$

$$\begin{cases} a+b+c=-3 \\ 27a+9b+3c=-21 \\ 64a+16b+4c=-24 \end{cases} \longrightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 27 & 9 & 3 & -21 \\ 64 & 16 & 4 & -24 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -18 & -24 & 60 \\ 0 & -48 & -60 & 168 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -3 & -4 & 10 \\ 0 & 4 & 5 & -14 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -3 & -4 & 10 \\ 0 & 1 & 1 & -4 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \\ 0 & 1 & 1 & -4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & -6 \end{array}\right] \longrightarrow$$

$\boxed{a=1}$, $\boxed{b=-6}$, $\boxed{c=2}$

39.) Since $\left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & 0 \\ a_2 & b_2 & c_2 & 0 \\ a_3 & b_3 & c_3 & 0 \end{array}\right] \sim \cdots \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$,

then

$$\left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & 3 \\ a_2 & b_2 & c_2 & 7 \\ a_3 & b_3 & c_3 & 11 \end{array}\right] \sim \cdots \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array}\right]$$ so exactly one solution

41.) a.) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$, $\begin{bmatrix}1&0&*\\0&1&*\\0&0&0\end{bmatrix}$, $\begin{bmatrix}1&*&0\\0&0&1\\0&0&0\end{bmatrix}$,

$\begin{bmatrix}1&*&*\\0&0&0\\0&0&0\end{bmatrix}$, $\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}$, $\begin{bmatrix}0&1&*\\0&0&0\\0&0&0\end{bmatrix}$, $\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}$

b.) $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$, $\begin{bmatrix}1&0&0&*\\0&1&0&*\\0&0&1&*\\0&0&0&0\end{bmatrix}$, $\begin{bmatrix}1&0&*&0\\0&1&*&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$,

$\begin{bmatrix}1&*&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$, $\begin{bmatrix}1&*&*&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}$, $\begin{bmatrix}1&0&*&*\\0&1&*&*\\0&0&0&0\\0&0&0&0\end{bmatrix}$,

$\begin{bmatrix}1&*&0&*\\0&0&1&*\\0&0&0&0\\0&0&0&0\end{bmatrix}$, $\begin{bmatrix}1&*&*&*\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$, $\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$,

$\begin{bmatrix}0&1&0&*\\0&0&1&*\\0&0&0&0\\0&0&0&0\end{bmatrix}$, $\begin{bmatrix}0&1&*&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}$, $\begin{bmatrix}0&1&*&*\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$,

$\begin{bmatrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}$, $\begin{bmatrix}0&0&1&*\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$, $\begin{bmatrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$

43.) a.) Assume that $ad-bc \neq 0$.

case 1: assume both $a=0$ and $c=0$. Then $ad-bc = 0-0=0$, so this case is impossible

case 2: Assume $a \neq 0$. Then

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \sim \begin{bmatrix} 1 & b/a \\ c & d \end{bmatrix} \sim \begin{bmatrix} 1 & b/a \\ 0 & -c \cdot b/a + d \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & b/a \\ 0 & -\frac{bc}{a} + \frac{ad}{a} \end{bmatrix} \sim \begin{bmatrix} 1 & b/a \\ 0 & \frac{ad-bc}{a} \end{bmatrix}$$

(where $\frac{ad-bc}{a} \neq 0$)

$$\sim \begin{bmatrix} 1 & b/a \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

case 3: Assume $c \neq 0$. Then

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \sim \begin{bmatrix} a & b \\ 1 & d/c \end{bmatrix} \sim \begin{bmatrix} 0 & -a \cdot \frac{d}{c} + b \\ 1 & d/c \end{bmatrix}$$

$$\sim \begin{bmatrix} 0 & -\frac{ad}{c} + \frac{bc}{c} \\ 1 & d/c \end{bmatrix} \sim \begin{bmatrix} 0 & \frac{bc-ad}{c} \\ 1 & d/c \end{bmatrix}$$

(where $\frac{bc-ad}{c} \neq 0$)

$$\sim \begin{bmatrix} 0 & 1 \\ 1 & d/c \end{bmatrix} \sim \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

b.) If $\begin{cases} ax+by=k \\ cx+dy=l \end{cases}$ then

$$\left[\begin{array}{cc|c} a & b & k \\ c & d & l \end{array}\right] \sim \cdots \sim \left[\begin{array}{cc|c} 1 & 0 & m \\ 0 & 1 & n \end{array}\right]$$ so the system has <u>exactly</u> one solution

TRUE/FALSE

(a) T (b) F (c) F (d) T

(e) T (f) F (g) T (h) F

(i) F