# Section 6.4

3.) $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 3 & 5 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$,

$$A^TA = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 21 & 25 \\ 25 & 35 \end{bmatrix},$$

$$A^T\vec{b} = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \end{bmatrix}; \text{ now solve}$$

$$A^TA\vec{x} = A^T\vec{b} \Rightarrow$$

$$\begin{bmatrix} 21 & 25 \\ 25 & 35 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \end{bmatrix} \Rightarrow \begin{matrix} x & y & = \\ \end{matrix} \left[\begin{array}{cc|c} 21 & 25 & 20 \\ 25 & 35 & 20 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 126 & 150 & 120 \\ 125 & 175 & 100 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -25 & 20 \\ 125 & 175 & 100 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & -25 & 20 \\ 0 & 3300 & -2400 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -25 & 20 \\ 0 & 1 & -8/11 \end{array}\right]$$

$$\begin{matrix} x & y & = \\ \end{matrix}$$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 20/11 \\ 0 & 1 & -8/11 \end{array}\right] \Rightarrow \boxed{x = \frac{20}{11}}, \boxed{y = \frac{-8}{11}};$$

$$\text{proj}_W\vec{b} = A\vec{x} = \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 20/11 \\ -8/11 \end{bmatrix} = \boxed{\begin{bmatrix} 28/11 \\ 16/11 \\ 40/11 \end{bmatrix}};$$

and least squares error is

$$\|\vec{b} - A\vec{x}\| = \left\| \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} - \begin{bmatrix} 28/11 \\ 16/11 \\ 40/11 \end{bmatrix} \right\| = \left\| \begin{bmatrix} -6/11 \\ -27/11 \\ 15/11 \end{bmatrix} \right\|$$

$$= \sqrt{\left(\frac{-6}{11}\right)^2 + \left(\frac{-27}{11}\right)^2 + \left(\frac{15}{11}\right)^2} = \sqrt{\frac{990}{121}}$$

$$= \frac{3}{11}\sqrt{110}$$

4.) $A = \begin{bmatrix} 2 & -2 \\ 1 & 1 \\ 3 & 1 \end{bmatrix}$, $A^T = \begin{bmatrix} 2 & 1 & 3 \\ -2 & 1 & 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$,

$$A^TA = \begin{bmatrix} 2 & 1 & 3 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 1 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 0 \\ 0 & 6 \end{bmatrix},$$

$$A^T\vec{b} = \begin{bmatrix} 2 & 1 & 3 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}; \text{ now solve}$$

$$A^TA\vec{x} = A^T\vec{b} \Rightarrow$$

$$\begin{bmatrix} 14 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} & & \\ 14 & 0 & 6 \\ 0 & 6 & -4 \end{array}\right] \quad \begin{matrix} & & x & y & = \end{matrix}$$

$$\begin{matrix} & x & y & = \\ \sim & \left[\begin{array}{cc|c} 1 & 0 & 3/7 \\ 0 & 1 & -2/3 \end{array}\right] \end{matrix} \Rightarrow \boxed{x = 3/7}, \boxed{y = -2/3};$$

$$\text{proj}_W \vec{b} = A\vec{x} = \begin{bmatrix} 2 & -2 \\ 1 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 3/7 \\ -2/3 \end{bmatrix}$$

$$= \begin{bmatrix} 6/7 + 4/3 \\ 3/7 - 2/3 \\ 9/7 - 2/3 \end{bmatrix} = \boxed{\begin{bmatrix} 46/21 \\ -5/21 \\ 13/21 \end{bmatrix}}; \text{ and least squares error is}$$

$$\|\vec{b} - A\vec{x}\| = \left\| \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} - \begin{bmatrix} 46/21 \\ -5/21 \\ 13/21 \end{bmatrix} \right\| = \left\| \begin{bmatrix} -4/21 \\ -16/21 \\ 8/21 \end{bmatrix} \right\|$$

$$= \sqrt{\left(\frac{-4}{21}\right)^2 + \left(\frac{-16}{21}\right)^2 + \left(\frac{8}{21}\right)^2} = \sqrt{\frac{336}{441}} = \frac{4}{21}\sqrt{21}$$

5.) $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -2 \\ 1 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ -1 & -2 & 0 & -1 \end{bmatrix}$,

$$\vec{b} = \begin{bmatrix} 6 \\ 0 \\ 9 \\ 3 \end{bmatrix}, \quad A^TA = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ -1 & -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -2 \\ 1 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 4 & -6 \\ 4 & 3 & -3 \\ -6 & -3 & 6 \end{bmatrix}, \quad A^T\vec{b} = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ -1 & -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ 9 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 12 \\ -9 \end{bmatrix}; \text{ now solve}$$

$$A^TA\vec{x} = A^T\vec{b} \Rightarrow$$

$$\begin{bmatrix} 7 & 4 & -6 \\ 4 & 3 & -3 \\ -6 & -3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 12 \\ -9 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} & & & x\ y\ z = \\ 7 & 4 & -6 & 18 \\ 4 & 3 & -3 & 12 \\ -6 & -3 & 6 & -9 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 9 \\ 4 & 3 & -3 & 12 \\ -6 & -3 & 6 & -9 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 9 \\ 0 & -1 & -3 & -24 \\ 0 & 3 & 6 & 45 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & -15 \\ 0 & 1 & 3 & 24 \\ 0 & 0 & -3 & -27 \end{array}\right] \sim \left[\begin{array}{ccc|c} & & & x\ y\ z = \\ 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

$\Rightarrow$ $\boxed{x=12}$, $\boxed{y=-3}$, $\boxed{z=9}$;

$$\text{proj}_W \vec{b} = A\vec{x} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -2 \\ 1 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 12 \\ -3 \\ 9 \end{bmatrix} = \boxed{\begin{bmatrix} 3 \\ 3 \\ 9 \\ 0 \end{bmatrix}};$$

and least squares error is:

$$\|\vec{b} - A\vec{x}\| = \left\| \begin{bmatrix} 6 \\ 0 \\ 9 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \\ 9 \\ 0 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 3 \\ -3 \\ 0 \\ 3 \end{bmatrix} \right\|$$

$$= \sqrt{3^2 + (-3)^2 + (0)^2 + (3)^2} = \sqrt{27} = 3\sqrt{3}$$

12.) $A = \begin{bmatrix} 1 & 3 \\ -2 & -6 \\ 3 & 9 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & -2 & 3 \\ 3 & -6 & 9 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$,

$$A^TA = \begin{bmatrix} 1 & -2 & 3 \\ 3 & -6 & 9 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & -6 \\ 3 & 9 \end{bmatrix} = \begin{bmatrix} 14 & 42 \\ 42 & 126 \end{bmatrix},$$

$$A^T\vec{b} = \begin{bmatrix} 1 & -2 & 3 \\ 3 & -6 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \end{bmatrix}; \text{ now solve}$$

$$A^T A\vec{x} = A^T\vec{b} \Rightarrow$$

$$\begin{bmatrix} 14 & 42 \\ 42 & 126 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} x & y & = \\ 14 & 42 & 4 \\ 42 & 126 & 12 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} x & y & = \\ 1 & 3 & 2/7 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow x + 3y = 2/7, \text{ so}$$

let $\boxed{y = t \text{ any } \#}$ $\Rightarrow$ $\boxed{x = \frac{2}{7} - 3t}$;

$$\text{proj}_W \vec{b} = A\vec{x} = \begin{bmatrix} 1 & 3 \\ -2 & -6 \\ 3 & 9 \end{bmatrix}\begin{bmatrix} 2/7 - 3t \\ t \end{bmatrix}$$

$$= \begin{bmatrix} 2/7 - 3t + 3t \\ -4/7 + 6t - 6t \\ 6/7 - 9t + 9t \end{bmatrix} = \boxed{\begin{bmatrix} 2/7 \\ -4/7 \\ 6/7 \end{bmatrix}};$$

and least squares error is

$$\|\vec{b} - A\vec{x}\| = \left\| \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 2/7 \\ -4/7 \\ 6/7 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 5/7 \\ 4/7 \\ 1/7 \end{bmatrix} \right\|$$

$$= \sqrt{\left(\frac{5}{7}\right)^2 + \left(\frac{4}{7}\right)^2 + \left(\frac{1}{7}\right)^2} = \sqrt{\frac{42}{49}} = \frac{\sqrt{42}}{7}$$

13.) $A = \begin{bmatrix} -1 & 3 & 2 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, $A^T = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 7 \\ 0 \\ -7 \end{bmatrix}$,

$$A^TA = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} -1 & 3 & 2 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 4 \\ -1 & 11 & 10 \\ 4 & 10 & 14 \end{bmatrix},$$

$$A^T\vec{b} = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 0 \\ -7 \end{bmatrix} = \begin{bmatrix} -7 \\ 14 \\ 7 \end{bmatrix};$$ now solve

$$A^TA\vec{x} = A^T\vec{b} \Rightarrow$$

$$\begin{bmatrix} 5 & -1 & 4 \\ -1 & 11 & 10 \\ 4 & 10 & 14 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 14 \\ 7 \end{bmatrix} \Rightarrow \begin{array}{c} x\ y\ z = \\ \left[\begin{array}{ccc|c} 5 & -1 & 4 & -7 \\ -1 & 11 & 10 & 14 \\ 4 & 10 & 14 & 7 \end{array}\right] \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 0 & 54 & 54 & 63 \\ 1 & -11 & -10 & -14 \\ 0 & 54 & 54 & 63 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 1 & 1 & 7/6 \\ 1 & -11 & -10 & -14 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{c} x\ y\ z = \\ \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & -7/6 \\ 0 & 1 & 1 & 7/6 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \Rightarrow \begin{cases} x + z = -7/6 \\ y + z = 7/6, \end{cases}$$

so let $\boxed{z = t \text{ any } \#} \Rightarrow$

$\boxed{x = -t - 7/6}$, $\boxed{y = -t + 7/6}$ ;

$$\text{proj}_W \vec{b} = A\vec{x} = \begin{bmatrix} -1 & 3 & 2 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -t - 7/6 \\ -t + 7/6 \\ t \end{bmatrix}$$

$$= \begin{bmatrix} t + 7/6 - 3t + 21/6 + 2t \\ -2t - 14/6 - t + 7/6 + 3t \\ -t + 7/6 + t \end{bmatrix} = \boxed{\begin{bmatrix} 14/3 \\ -7/6 \\ 7/6 \end{bmatrix}} ;$$

and least squares error is

$$\|\vec{b} - A\vec{x}\| = \left\| \begin{bmatrix} 7 \\ 0 \\ -7 \end{bmatrix} - \begin{bmatrix} 14/3 \\ -7/6 \\ 7/6 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 7/3 \\ 7/6 \\ -49/6 \end{bmatrix} \right\|$$

$$= \sqrt{\left(\frac{7}{3}\right)^2 + \left(\frac{7}{6}\right)^2 + \left(\frac{-49}{6}\right)^2}$$

$$= \sqrt{\frac{196}{36} + \frac{49}{36} + \frac{2401}{36}} = \sqrt{\frac{2646}{36}}$$

$$= \sqrt{\frac{9 \cdot 294}{9 \cdot 4}} = \sqrt{\frac{147}{2}}$$

16.) $A = \begin{bmatrix} 5 & 1 \\ 1 & 3 \\ 4 & -2 \end{bmatrix}$, $A^T = \begin{bmatrix} 5 & 1 & 4 \\ 1 & 3 & -2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -4 \\ 2 \\ 3 \end{bmatrix}$,

$$A^TA = \begin{bmatrix} 5 & 1 & 4 \\ 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 1 & 3 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 42 & 0 \\ 0 & 14 \end{bmatrix},$$

$$A^T\vec{b} = \begin{bmatrix} 5 & 1 & 4 \\ 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -6 \\ -4 \end{bmatrix}; \text{ now solve}$$

$$A^TA\vec{x} = A^T\vec{b} \Rightarrow \begin{bmatrix} 42 & 0 \\ 0 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -4 \end{bmatrix} \Rightarrow$$

$$\begin{array}{cc} x & y \end{array} =$$

$$\left[\begin{array}{cc|c} 42 & 0 & -6 \\ 0 & 14 & -4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & -1/7 \\ 0 & 1 & -2/7 \end{array}\right] \Rightarrow$$

$\boxed{x = -1/7}$, $\boxed{y = -2/7}$; then

$$\text{proj}_W \vec{b} = A\vec{x} = \begin{bmatrix} 5 & 1 \\ 1 & 3 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} -1/7 \\ -2/7 \end{bmatrix} = \boxed{\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}};$$

By Theorem 6.4.4.:

$$\text{proj}_W \vec{b} = A(A^TA)^{-1}A^T\vec{b}$$

$$= \begin{bmatrix} 5 & 1 \\ 1 & 3 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 42 & 0 \\ 0 & 14 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 5 & 1 & 4 \\ 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 1 \\ 1 & 3 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1/42 & 0 \\ 0 & 1/14 \end{bmatrix} \cdot \begin{bmatrix} -6 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 1 \\ 1 & 3 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} -1/7 \\ -2/7 \end{bmatrix} = \boxed{\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}}$$

17.) Solve $A\vec{x} = \vec{b}$, where $A = \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix}$,

$A^T = \begin{bmatrix} -1 & 2 & 1 \\ 2 & 2 & 4 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ -6 \\ 1 \end{bmatrix}$;

$$A^TA = \begin{bmatrix} -1 & 2 & 1 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & 24 \end{bmatrix},$$

$$A^T\vec{b} = \begin{bmatrix} -1 & 2 & 1 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -6 \\ 1 \end{bmatrix} = \begin{bmatrix} -12 \\ -6 \end{bmatrix}; \text{ now solve}$$

$$A^TA\vec{x} = A^T\vec{b} \Rightarrow \begin{bmatrix} 6 & 6 \\ 6 & 24 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ -6 \end{bmatrix} \Rightarrow$$

$$\begin{matrix} & x & y & = \\ \left[\begin{array}{cc|c} 6 & 6 & -12 \\ 6 & 24 & -6 \end{array}\right] & & & \sim \left[\begin{array}{cc|c} 1 & 1 & -2 \\ 6 & 24 & -6 \end{array}\right] \end{matrix}$$

$$\sim \left[\begin{array}{cc|c} 1 & 1 & -2 \\ 0 & 18 & 6 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & -2 \\ 0 & 1 & 1/3 \end{array}\right]$$

$$\begin{matrix} & x & y & = \\ \sim \left[\begin{array}{cc|c} 1 & 0 & -2/3 \\ 0 & 1 & 1/3 \end{array}\right] & & & \Rightarrow \boxed{x = -7/3}, \boxed{y = 1/3}; \text{ then} \end{matrix}$$

$$\text{proj}_W \vec{b} = A\vec{x} = \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -7/3 \\ 1/3 \end{bmatrix} = \boxed{\begin{bmatrix} 3 \\ -4 \\ -1 \end{bmatrix}}$$

19.) a basis for the x-axis is $\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$ so let $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow$ standard matrix for the orthogonal projection is

$$P = A(A^TA)^{-1}A^T$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left( \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} [1]^{-1} [1 \ 0] = \begin{bmatrix} 1 \\ 0 \end{bmatrix} [1 \ 0] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

21.) a basis for the $xz$-plane is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ so let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow$ standard matrix for the orthogonal projection is

$$P = A(A^T A)^{-1} A^T$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \left( \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

22.) a basis for the yz-plane is $\left\{\begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$ so let $A = \begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix} \Rightarrow$ standard matrix for the orthogonal projection is

$$P = A(A^TA)^{-1}A^T$$

$$= \begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix}\left(\begin{bmatrix}0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix}\right)^{-1}\begin{bmatrix}0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}^{-1}\begin{bmatrix}0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix}\begin{bmatrix}0&1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

26.) Line $L: \begin{cases} x = 2t \\ y = -t \\ z = 4t \end{cases}$

a.) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2t \\ -t \\ 4t \end{bmatrix} = t\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ so basis for line is $\left\{\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}\right\}$

b.) Let $A = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$, then standard matrix for the orthogonal projection is

$$P = A(A^TA)^{-1}A^T$$

$$= \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}\left(\begin{bmatrix} 2 & -1 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}\right)^{-1}\begin{bmatrix} 2 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}\begin{bmatrix} 21 \end{bmatrix}^{-1}\begin{bmatrix} 2 & -1 & 4 \end{bmatrix}$$

$$= \left(\tfrac{1}{21}\right)\begin{bmatrix} 4 & -2 & 8 \\ -2 & 1 & -4 \\ 8 & -4 & 16 \end{bmatrix} = \begin{bmatrix} 4/21 & -2/21 & 8/21 \\ -2/21 & 1/21 & -4/21 \\ 8/21 & -4/21 & 16/21 \end{bmatrix}$$

# TRUE/FALSE

(a) T (b) F (c) T (d) T

(e) F (f) T (g) F (h) T