Section 1.3

1.) a.) NO b.) YES, 4X4 c.) YES, 4X2
d.) YES, 5X2 e.) YES, 4X5
f.) YES, 5X5

2.) a.) YES, 5X4 b.) NO c.) YES, 4X2
d.) YES, 2X4 e.) YES, 5X2
f.) NO

4.) a.) $2A^T + C = 2\begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 6 & -2 & 2 \\ 0 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix}$

b.) $D^T - E^T = \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$

c.) $(D-E)^T = \begin{bmatrix} -5 & 4 & -1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}^T = \begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$

d.) NO e.) $\frac{1}{2}C^T - \frac{1}{4}A$

$= \frac{1}{2}\begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} - \frac{1}{4}\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 3/2 \\ 2 & 1/2 \\ 1 & 5/2 \end{bmatrix} - \begin{bmatrix} 3/4 & 0 \\ -1/4 & 1/2 \\ 1/4 & 1/4 \end{bmatrix}$

$$= \begin{bmatrix} -1/4 & 3/2 \\ 9/4 & 0 \\ 3/4 & 9/4 \end{bmatrix}$$

f.) $B - B^T = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

g.) $2E^T - 3D^T = 2\begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 12 & -2 & 8 \\ 2 & 2 & 2 \\ 6 & 4 & 6 \end{bmatrix} - \begin{bmatrix} 3 & -3 & 9 \\ 15 & 0 & 6 \\ 6 & 3 & 12 \end{bmatrix} = \begin{bmatrix} 9 & 1 & -1 \\ -13 & 2 & -4 \\ 0 & 1 & -6 \end{bmatrix}$$

h.) $(2E^T - 3D^T)^T = \begin{bmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -6 \end{bmatrix}$

i.) $(CD)E = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \cdot E$

$$= \begin{bmatrix} 3 & 9 & 14 \\ -17 & 25 & 27 \end{bmatrix} \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 65 & 26 & 69 \\ 185 & 69 & 182 \end{bmatrix}$$

j.) NO

k.) $DE^T = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 17 & 8 & 15 \\ -3 & 3 & -1 \\ 32 & 7 & 26 \end{bmatrix} \quad \text{so } tr(DE^T) = 17 + 3 + 26 = 46$$

l.) NO

7.) a.) $\begin{bmatrix} 67 & 41 & 41 \end{bmatrix}$

b.) $\begin{bmatrix} 63 & 67 & 57 \end{bmatrix}$

c.) $\begin{bmatrix} 41 \\ 21 \\ 67 \end{bmatrix}$ d.) $\begin{bmatrix} 6 \\ 6 \\ 63 \end{bmatrix}$

~~e.) [24 56 97]~~ f.) $\begin{bmatrix} 76 \\ 98 \\ 97 \end{bmatrix}$

9.) a.) column 1 of AA:

$$3\begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} + 6\begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 0\begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix} = \begin{bmatrix} -3 \\ 48 \\ 24 \end{bmatrix}$$

column 2 of AA:

$$-2\begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} + 5\begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 4\begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix} = \begin{bmatrix} 12 \\ 29 \\ 56 \end{bmatrix}$$

column 3 of AA:

$$7\begin{bmatrix}3\\6\\0\end{bmatrix}+4\begin{bmatrix}-2\\5\\4\end{bmatrix}+9\begin{bmatrix}7\\4\\9\end{bmatrix}=\begin{bmatrix}76\\98\\97\end{bmatrix}$$

11.) a.)

$$\underset{A}{\begin{bmatrix}2&-3&5\\9&-1&1\\1&5&4\end{bmatrix}}\underset{X}{\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}}=\underset{b}{\begin{bmatrix}7\\-1\\0\end{bmatrix}}$$

b.)

$$\underset{A}{\begin{bmatrix}4&0&-3&1\\5&1&0&-8\\2&-5&9&-1\\0&3&-1&7\end{bmatrix}}\underset{X}{\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}}=\underset{b}{\begin{bmatrix}1\\3\\0\\2\end{bmatrix}}$$

13.) b.)

$$\begin{cases}x+y+z=2\\2x+3y=2\\5x-3y-6z=-9\end{cases}$$

16.)

$$\cancel{\begin{bmatrix}2&2&k\end{bmatrix}}\begin{bmatrix}1&2&0\\2&0&3\\0&3&1\end{bmatrix}\begin{bmatrix}2\\2\\k\end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4+3k & 6+k \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix}$$

$$= 12 + (8+6k) + (6k + k^2)$$

$$= k^2 + 12k + 20 = 0 \longrightarrow$$

$$(k+10)(k+2) = 0 \rightarrow \boxed{k = -10}$$

or $\boxed{k = -2}$

18.) $\begin{bmatrix} 0 & -2 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 1 & 4 & 1 \\ -3 & 0 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 0 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 4 & 1 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} \begin{bmatrix} -3 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 4 & 16 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 0 & -4 \\ 9 & 0 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & -4 \\ 13 & 16 & -2 \end{bmatrix}$$

19.) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} \begin{bmatrix} 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 6 \end{bmatrix} \begin{bmatrix} 5 & 6 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 6 & 8 \\ 15 & 20 \end{bmatrix} + \begin{bmatrix} 15 & 18 \\ 30 & 36 \end{bmatrix}$

$= \begin{bmatrix} 22 & 28 \\ 49 & 64 \end{bmatrix}$

23.) $\begin{bmatrix} a & 3 \\ -1 & a+b \end{bmatrix} = \begin{bmatrix} 4 & d-2c \\ d+2c & -2 \end{bmatrix} \rightarrow$

$\left\{ \begin{array}{l} \boxed{a = 4} \\ \left. \begin{array}{l} d - 2c = 3 \\ d + 2c = -1 \end{array} \right\} \rightarrow 2d = 2 \rightarrow \boxed{d = 1} \rightarrow \boxed{c = -1} \\ a + b = -2 \rightarrow (4) + b = -2 \rightarrow \boxed{b = -6} \end{array} \right.$

24.) $\begin{bmatrix} a-b & b+a \\ 3d+c & 2d-c \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 6 \end{bmatrix} \rightarrow$

$\left\{ \begin{array}{l} \left. \begin{array}{l} a - b = 8 \\ b + a = 1 \end{array} \right\} \rightarrow 2a = 9 \rightarrow \boxed{a = \frac{9}{2}} \rightarrow \boxed{b = \frac{-7}{2}} \\ \left. \begin{array}{l} 3d + c = 7 \\ 2d - c = 6 \end{array} \right\} \rightarrow 5d = 13 \rightarrow \boxed{d = 13/5} \rightarrow \end{array} \right.$

$c = 2\left(\frac{13}{5}\right) - \frac{30}{5} \rightarrow \boxed{c = \frac{-4}{5}}$

25.) assume A is $m \times r$ and B is $r \times n$, so AB is $m \times n$.

a.) If the kth row of A is all zeroes then so is the kth row of AB.

b.) If the kth column of B is all zeroes, then so is the kth column of AB.

26.) b.)

$a_{ij} = 0$ if $i > j$

row

column

$$\begin{bmatrix} * & * & * & * & * & * \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

d.)

$a_{ij} = 0$

if $|i-j| > 1$

difference is 2 or more

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

27.) $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ x-y \\ 0 \end{bmatrix} \longrightarrow$

$$\begin{cases} ax+by+cz = x+y \\ dx+ey+fz = x-y \\ gx+hy+iz = 0 \end{cases} \longrightarrow$$

$\boxed{a=1}$, $\boxed{b=1}$, $\boxed{c=0}$,

$\boxed{d=1}$, $\boxed{e=-1}$, $\boxed{f=0}$,

$\boxed{g=0}$, $\boxed{h=0}$, $\boxed{i=0}$

28.) $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} xy \\ 0 \\ 0 \end{bmatrix} \longrightarrow$

$$\begin{cases} ax+by+cz = xy \\ dx+ey+fz = 0 \\ gx+hy+iz = 0 \end{cases}$$

unsolvable since $a \neq y$ and $b \neq x$ since a, b are constants and x, y are variables

29.) Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

a.) $B^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, but by easy guess-and-check both $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$ work.

b.) $B^2 = \begin{bmatrix} 5 & 0 \\ 0 & 9 \end{bmatrix}$ is solved by

$B = \begin{bmatrix} \sqrt{5} & 0 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -\sqrt{5} & 0 \\ 0 & -3 \end{bmatrix}$,

$B = \begin{bmatrix} -\sqrt{5} & 0 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} \sqrt{5} & 0 \\ 0 & -3 \end{bmatrix}$

c.) Not all 2x2 matrices have square roots:

$B^2 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$ (SEE part a.)

$$\begin{cases} a^2+bc = -1 \\ ab+bd = b(a+d) = 0 \rightarrow b=0 \text{ or } a=-d \\ ac+cd = c(a+d) = 0 \rightarrow c=0 \text{ or } a=-d \\ bc+d^2 = 0 \end{cases}$$

case 1: If $b=0$, then
$-1 = a^2+bc = a^2+(0)c = a^2$,
so this is impossible!

case 2: If $c=0$, then
$-1 = a^2+bc = a^2+b(0) = a^2$,
so this is impossible.

case 3: If $a=-d$, then
$\begin{cases} a^2+bc=-1 \text{ and} \\ d^2+bc = (-a)^2+bc = a^2+bc = 0, \end{cases}$
so this is impossible.

$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$ has NO square root.

30.) a.) YES, let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then

$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

b.) YES, let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then

$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A$

32.) b.) $a_{ij} = i^{j-1}$; $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix}$

i: row, j: column

33.) $\underset{\text{(units)}}{\begin{bmatrix} 3 & 4 & 3 \\ 5 & 6 & 0 \\ 2 & 9 & 4 \\ 1 & 1 & 7 \end{bmatrix}} \underset{(\$)}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} = \begin{bmatrix} 3+8+9 \\ 5+12+0 \\ 2+18+12 \\ 1+2+21 \end{bmatrix} = \begin{bmatrix} 20 \\ 17 \\ 32 \\ 24 \end{bmatrix} \rightarrow$

→ 20: January revenue,
17: February revenue,
32: March revenue,
24: April revenue

36.) a.) Assume A is $m \times n$. If AB is defined, then B is $n \times k$ and

$$AB \text{ is } m \times k.$$

If BA is defined, then it must be that

$$k = m.$$

Thus, AB is $m \times m$ (square)
and BA is $n \times n$ (square)

TRUE/FALSE

(a) T (b) F (c) F (d) F

(e) T (f) F (g) F (h) T

(i) T (j) T (k) T (l) F

(m) T (n) T (o) F