Section 3.1

1.) a.) $\vec{v} = \overrightarrow{(4-1, 1-5)} = \overrightarrow{(3, -4)}$

b.) $\vec{v} = \overrightarrow{(0-2, 0-3, 4-0)} = \overrightarrow{(-2, -3, 4)}$

2.) a.) $\vec{v} = \overrightarrow{(-3-2, 3-3)} = \overrightarrow{(-5, 0)}$

b.) $\vec{v} = \overrightarrow{(0-3, 4-0, 4-4)} = \overrightarrow{(-3, 4, 0)}$

3.) a.) $\overrightarrow{P_1P_2} = \overrightarrow{(2-3, 8-5)} = \overrightarrow{(-1, 3)}$

b.) $\overrightarrow{P_1P_2} = \overrightarrow{(2-5, 4-(-2), 2-1)} = \overrightarrow{(-3, 6, 1)}$

6.) a.) Let $A = (a, b)$, then
$\overrightarrow{(1, 2)} = \overrightarrow{AB} = \overrightarrow{(2-a, 0-b)} = \overrightarrow{(2-a, -b)}$, so
$1 = 2-a \Rightarrow a = 1$; $-b = 2 \Rightarrow b = -2$, and
initial point: $A = (1, -2)$

b.) Let $B = (a, b, c)$, then
$\overrightarrow{(1, 1, 3)} = \overrightarrow{AB} = \overrightarrow{(a-0, b-2, c-0)} = \overrightarrow{(a, b-2, c)}$,
so $a = 1$, $b-2 = 1 \Rightarrow b = 3$, $c = 3 \Rightarrow$
terminal point: $B = (1, 3, 3)$

8.) Let $Q = (a, b, c)$, then

$\overrightarrow{PQ} = \overrightarrow{(-1-a, 3-b, -5-c)}$

a.) $\overrightarrow{(6,7,-3)} = \overrightarrow{(-1-a, 3-b, -5-c)}$, so

$6 = -1-a \Rightarrow a = -7$, $7 = 3-b \Rightarrow b = -4$,

$-3 = -5-c \Rightarrow c = -2$; and

terminal point: $Q = (-7, -4, -2)$

b.) $\overrightarrow{(-6,-7,3)} = \overrightarrow{(-1-a, 3-b, -5-c)}$, so

$-6 = -1-a \Rightarrow a = 5$; $-7 = 3-b \Rightarrow b = 10$;

$3 = -5-c \Rightarrow c = -8$; and

terminal point: $Q = (5, 10, -8)$

9.) a.) $\vec{u} + \vec{w} = \overrightarrow{(4,-1)} + \overrightarrow{(-3,-3)} = \overrightarrow{(1,-4)}$

b.) $\vec{v} - 3\vec{u} = \overrightarrow{(0,5)} - 3\overrightarrow{(4,-1)} = \overrightarrow{(0,5)} - \overrightarrow{(12,-3)}$

$= \overrightarrow{(-12, 8)}$

c.) $2(\vec{u} - 5\vec{w}) = 2\overrightarrow{(4,-1)} - 10\overrightarrow{(-3,-3)}$

$= \overrightarrow{(8,-2)} - \overrightarrow{(-30,-30)} = \overrightarrow{(38, 28)}$

d.) $3\vec{v} - 2(\vec{u} + 2\vec{w}) = 3\overrightarrow{(0,5)} - 2\overrightarrow{(4,-1)} - 4\overrightarrow{(-3,-3)}$

$= \overrightarrow{(0,15)} - \overrightarrow{(8,-2)} - \overrightarrow{(-12,-12)}$

$= \overrightarrow{(-8,17)} + \overrightarrow{(12,12)} = \overrightarrow{(4, 29)}$

10.) a.) $\vec{v} - \vec{w} = \overrightarrow{(4,0,-8)} - \overrightarrow{(6,-1,-4)}$

$= \overrightarrow{(-2,1,-4)}$

11.) b.) $-\vec{u} + \vec{v} - 4\vec{w} = -\overrightarrow{(-3,2,1,0)} + \overrightarrow{(4,7,-3,2)} - 4\overrightarrow{(5,-2,8,1)}$

$= \overrightarrow{(3,-2,-1,0)} + \overrightarrow{(4,7,-3,2)} - \overrightarrow{(20,-8,32,4)}$

$= \overrightarrow{(7,5,-4,2)} + \overrightarrow{(-20,8,-32,-4)}$

$= \overrightarrow{(-13,13,-36,-2)}$

14.) $2\vec{u} - \vec{v} + \vec{x} = 7\vec{x} + \vec{w} \Rightarrow$

$6\vec{x} = 2\vec{u} - \vec{v} - \vec{w}$

$= 2\overrightarrow{(1,2,-3,5,0)} - \overrightarrow{(0,4,-1,1,2)} - \overrightarrow{(7,1,-4,-2,3)}$

$= \overrightarrow{(2,4,-6,10,0)} + \overrightarrow{(0,-4,1,-1,-2)} + \overrightarrow{(-7,-1,4,2,-3)}$

$= \overrightarrow{(-5,-1,-1,11,-5)} \Rightarrow$

$\vec{x} = \frac{1}{6}\overrightarrow{(-5,-1,-1,11,-5)} = \overrightarrow{(-5/6, -1/6, -1/6, 11/6, -5/6)}$

15.) a.) NO

b.) YES: $\overrightarrow{(4,-2,0,-6,-10,-2)} = -2\overrightarrow{(-2,1,0,3,5,1)}$

c.) YES: $\overrightarrow{(0,0,0,0,0,0)} = 0\overrightarrow{(-2,1,0,3,5,1)}$

16.) a.) $\overrightarrow{(8t,-2)} = 2\overrightarrow{(4t,-1)}$, so $t=1$ works

b.) $\overrightarrow{(8t,2t)} = 2t\overrightarrow{(4,1)} \neq \overrightarrow{(4,-1)}$

no solution

c.) $4\overrightarrow{(1,t^2)} = \overrightarrow{(4,4t^2)} = \overrightarrow{(4,-1)} \Rightarrow$

$4t^2 = -1 \Rightarrow t^2 = \frac{-1}{4} \Rightarrow$ no solution

18.) $a\overrightarrow{(2,1,0,1,-1)} + b\overrightarrow{(-2,3,1,0,2)} = \overrightarrow{(-8,8,3,-1,7)} \Rightarrow$

$\overrightarrow{(2a,a,0,a,-a)} + \overrightarrow{(-2b,3b,b,0,2b)} = \overrightarrow{(-8,8,3,-1,7)} \Rightarrow$

$\overrightarrow{(2a-2b,a+3b,b,a,2b-a)} = \overrightarrow{(-8,8,3,-1,7)} \Rightarrow$

$$\begin{cases} 2a-2b=-8 \\ a+3b=8 \\ \boxed{b=3} \\ \boxed{a=-1} \\ 2b-a=7 \end{cases}$$

$\left.\begin{matrix} b=3 \\ a=-1 \end{matrix}\right\}$ satisfy all 5 equations

19.) $c_1\overrightarrow{(1,-1,0)} + c_2\overrightarrow{(3,2,1)} + c_3\overrightarrow{(0,1,4)} = \overrightarrow{(-1,1,19)} \Rightarrow$

$\overrightarrow{(c_1+3c_2,-c_1+2c_2+c_3,c_2+4c_3)} = \overrightarrow{(-1,1,19)} \Rightarrow$

$$\begin{cases} c_1+3c_2=-1 \\ -c_1+2c_2+c_3=1 \\ c_2+4c_3=19 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 0 & -1 \\ -1 & 2 & 1 & 1 \\ 0 & 1 & 4 & 19 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 0 & -1 \\ 0 & 5 & 1 & 0 \\ 0 & 1 & 4 & 19 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -12 & -58 \\ 0 & 0 & -19 & -95 \\ 0 & 1 & 4 & 19 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & -1 \end{array}\right] \Rightarrow c_1 = 2,\ c_2 = -1,\ c_3 = 5$$

20.) (SEE problem 19.)

$$\left[\begin{array}{ccc|c} -1 & 2 & 1 & -6 \\ 0 & 2 & -2 & 12 \\ 2 & -2 & 1 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & -1 & 6 \\ 0 & 1 & -1 & 6 \\ 0 & 2 & 3 & -8 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 18 \\ 0 & 1 & -1 & 6 \\ 0 & 0 & 5 & -20 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -4 \end{array}\right]$$

22.) (SEE problems 19 and 20.)

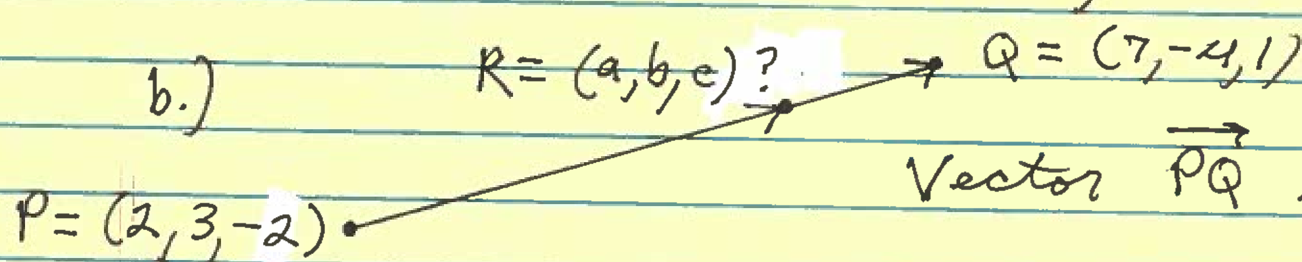
$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & -2 \\ 1 & -2 & 1 & 2 \\ 0 & 1 & 2 & 3 \end{array}\right] \Rightarrow 0 \cdot c_1 + 0 \cdot c_2 + 0 \cdot c_3 = -2$$

$\Rightarrow 0 = -2$ so NO Solution

23.) $P = (2, 3, -2)$, $Q = (7, -4, 1)$

a.) $\text{Midpoint} = \left(\frac{2+7}{2}, \frac{3+{}^{-}4}{2}, \frac{-2+1}{2}\right) = \left(\frac{9}{2}, \frac{-1}{2}, \frac{-1}{2}\right)$

b.)



Vector $\overrightarrow{PQ}$ is

$\overrightarrow{PQ} = \overrightarrow{(7-2, -4-3, 1-(-2))} = \overrightarrow{(5, -7, 3)}$; we want point R so that

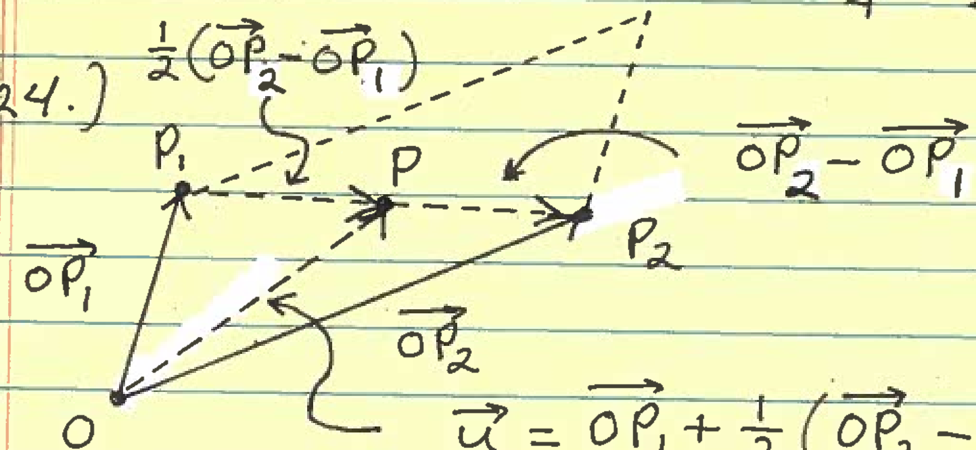
$$\overrightarrow{PR} = \frac{3}{4}\overrightarrow{PQ} \Rightarrow$$

$$\overrightarrow{(a-2, b-3, c+2)} = \frac{3}{4}\overrightarrow{(5, -7, 3)} \Rightarrow$$

$$\overrightarrow{(a-2, b-3, c+2)} = \overrightarrow{\left(15/4, -21/4, 9/4\right)} \Rightarrow$$

$$\begin{cases} a-2 = 15/4 \\ b-3 = -21/4 \\ c+2 = 9/4 \end{cases} \Rightarrow \begin{cases} a = \frac{15}{4}+\frac{8}{4} = 23/4 \\ b = \frac{12}{4} - \frac{21}{4} = -9/4 \\ c = \frac{9}{4} - \frac{8}{4} = 1/4 \end{cases}$$

24.)



$\vec{u} = \overrightarrow{OP_1} + \frac{1}{2}(\overrightarrow{OP_2} - \overrightarrow{OP_1})$, where point P is the MIDPOINT of line segment P_1P_2

25.) a.) $\vec{u} + \vec{v} + \vec{w} = \overrightarrow{(-2, 5)}$

b.) (Translate initial point $(-5, 4)$ to the origin.)

$$\vec{u}+\vec{v}+\vec{w} = \overrightarrow{(-2,-4)} + \overrightarrow{(5,-4)}$$
$$= \overrightarrow{(3,-8)}$$

26.) a.) $\vec{u}-\vec{v}+\vec{w} = (\vec{u}+\vec{v}+\vec{w}) - 2\vec{v}$

$$= \overrightarrow{(-2,5)} - 2\overrightarrow{(-10,2)}$$
$$= \overrightarrow{(-2,5)} + \overrightarrow{(20,-4)} = \overrightarrow{(18,1)}$$

b.) $\vec{u}-\vec{v}+\vec{w} = (\vec{u}+\vec{v}+\vec{w}) - 2\vec{v}$

$$= \overrightarrow{(-5,4)} - 2\overrightarrow{(-3,8)}$$
$$= \overrightarrow{(-5,4)} + \overrightarrow{(6,-16)} = \overrightarrow{(1,-12)}$$

27.)

$Q = (a,b,c)$?

$P = (1,3,7)$ $\quad R = (4,0,-6) \leftarrow$ (MIDPOINT)

$$\overrightarrow{PR} = \overrightarrow{(4-1,\ 0-3,\ -6-7)} = \overrightarrow{(3,-3,-13)} \Rightarrow$$
$$\overrightarrow{PQ} = 2\overrightarrow{PR} \Rightarrow$$
$$\overrightarrow{(a-1,\ b-3,\ c-7)} = 2\ \overrightarrow{(3,-3,-13)} \Rightarrow$$
$$\overrightarrow{(a-1,\ b-3,\ c-7)} = \overrightarrow{(6,-6,-26)} \Rightarrow$$

$$\begin{cases} a-1=6 \\ b-3=-6 \\ c-7=-26 \end{cases} \Rightarrow \begin{cases} a=7 \\ b=-3 \\ c=-19 \end{cases} \Rightarrow$$

$$Q=(7,-3,-19)$$

TRUE/FALSE

(a) F (b) F (c) F (d) T

(e) T (f) F (g) F (h) T

(i) F (j) T (k) F