

Two by Two

Linear Systems of Equations

Ex: Raw almonds selling for \$5/lb. are mixed with raw almonds selling for \$2/lb. How many pounds of each should be mixed to result in 100 pounds of almonds selling for \$4/lb.?

Let x : lbs. of \$5 almonds
 y : lbs. of \$2 almonds

then

(weight): $x + y = 100$ (lbs.)

(total cost): $\left(\frac{\$5}{\text{lb.}}\right)(x \text{ lbs.}) + \left(\frac{\$2}{\text{lb.}}\right)(y \text{ lbs.}) = \left(\frac{\$4}{\text{lb.}}\right)(100 \text{ lbs.})$

$$\rightarrow \begin{cases} x + y = 100 \\ 5x + 2y = 400 \end{cases}$$

$$\rightarrow \begin{cases} -5x + -5y = -500 \\ 5x + 2y = 400 \end{cases}$$

$$\rightarrow -3y = -100 \rightarrow y = \frac{100}{3} \text{ lbs.},$$

$$x = \frac{200}{3} \text{ lbs.}$$

Solving Two Equations, Two Unknowns

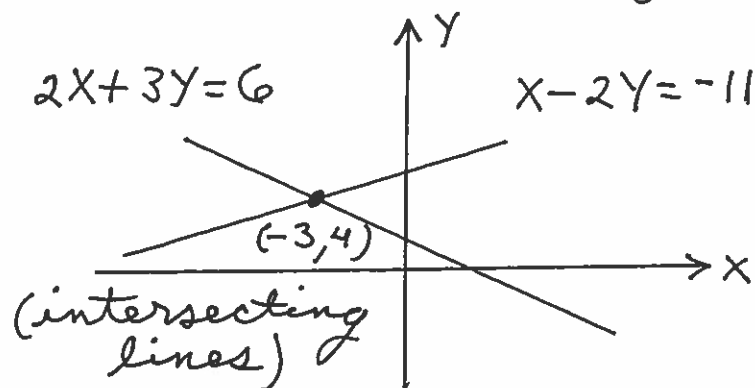
Ex: Solve the following systems.

$$1.) \quad \left. \begin{array}{l} x - 2y = -11 \\ 2x + 3y = 6 \end{array} \right\} \rightarrow \left. \begin{array}{l} -2x + 4y = 22 \\ 2x + 3y = 6 \end{array} \right\}$$

$$\rightarrow 7y = 28$$

$$\rightarrow \boxed{y = 4}, \boxed{x = -3}$$

ONE SOLUTION

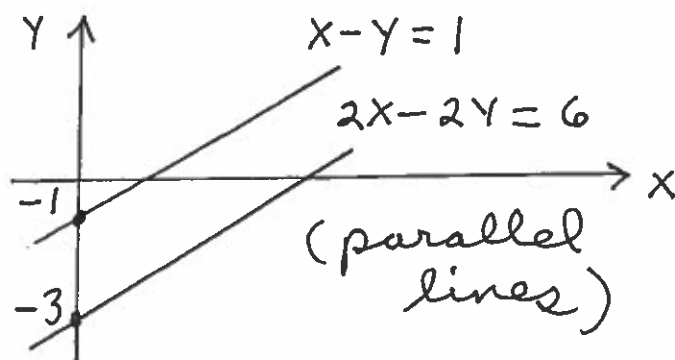


$$2.) \quad \left. \begin{array}{l} x - y = 1 \\ 2x - 2y = 6 \end{array} \right\} \rightarrow \left. \begin{array}{l} -2x + 2y = -2 \\ 2x - 2y = 6 \end{array} \right\}$$

$$\rightarrow 0 = 4 !!!$$

IMPOSSIBLE so

NO SOLUTION



$$3.) \quad \left. \begin{array}{l} x + 3y = -2 \\ -2x - 6y = 4 \end{array} \right\} \rightarrow \left. \begin{array}{l} 2x + 6y = -4 \\ -2x - 6y = 4 \end{array} \right\}$$

$$\rightarrow 0 = 0 \text{ (REDUNDANT)}; \text{ note that}$$

$$\left. \begin{array}{l} x + 3y = -2 \\ x + 3y = -2 \end{array} \right\}$$

(OVERDETERMINED
SYSTEM)

Then $x + 3y = -2 \rightarrow$

$x = -3y - 2$, so let $y = t$ any #

and $x = -3t - 2$;

INFINITELY MANY
SOLUTIONS

