Math 22A
Kouba
Determinants by Row Reduction

FACTS: Let A be an $n \times n$ matrix.

I.) If A has a row of zeroes or a column of zeroes, then $det(A) = 0$.

II.) Let A^T be the transpose of matrix A. Then $det(A) = det(A^T)$.

III.) If matrix A is diagonal, upper triangular, or lower triangular, then

$$det(A) = a_{11}a_{22}\cdots a_{nn}$$

(the product of the main diagonal entries)

THEOREM: Let A be an $n \times n$ matrix. If B is matrix which results from:

1.) a single row (or column) of matrix A being multiplied by a constant k, then

$$det(B) = k\ det(A)$$

2.) two rows (or columns) of matrix A are switched, then

$$det(B) = -det(A)$$

3.) a multiple of one row (or column) of matrix A being added to another row (or column) of A, then

$$det(B) = det(A)$$

(All of these facts are discussed in detail in the book with 3×3 examples.)

DETERMINANTS of ELEMENTARY MATRICES

RECALL: Let I be an $n \times n$ identity matrix. Then $det(I) = 1$ (the product of the main diagonal entries).

FACTS: Let E be an elementary matrix.

I.) If E multiplies a row by a nonzero constant k, then

$$det(E) = k$$

(the product of the main diagonal entries)

II.) If E adds a multiple of one row to another row, then

$$det(E) = +1$$

(the product of the main diagonal entries)

III.) If E switches rows, then

$$det(E) = -det(I) = -1$$

QUESTION: Why do Cofactor Expansion and Row Reduction result in the same determinant ? (See Section 2.3 and upcoming lecture.)

PROPERTIES of DETERMINANTS

RECALL: If E is an elementary matrix, then $det(E) = +1, -1,\ or\ k$ (the product of the main diagonal entries).

THEOREM A: If B is an $n \times n$ matrix and E is an elementary matrix, then

$$det(EB) = det(E)det(B)$$

PROOF: (You– Use previous results.)

THEOREM: If A is an $n \times n$ matrix and k a constant, then

$$det(kA) = k^n\ det(A)$$

PROOF: (You– Use row reduction properties.)

THEOREM N: A square matrix A is invertible iff $det(A) \neq 0$.

PROOF: (Me)

RECALL: Let A and B be $n \times n$ matrices. If A is not invertible, then AB is not invertible.

PROOF: (You)

THEOREM P: If A and B are $n \times n$ matrices, then

$$det(AB) = det(A)det(B)$$

PROOF: (Me)