Math 22A
Kouba
Translation Matrices in Three-Dimensional Space

PROBLEM: Suppose that we wish to translate all points (x, y, z) in 3D-Space in the following manner: Shift x-values a units; shift y-values b units; shift z-values c units. Let's first convert point (x, y, z) to vector form $\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$. Now consider the following translation matrix

$$T = \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Then

$$T \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x+a \\ y+b \\ z+c \\ 1 \end{pmatrix} .$$

Now convert this resulting vector to the point $(x + a, y + b, z + c)$.

EXAMPLE: Suppose that we wish to translate all points (x, y, z) in 3D-Space in the following manner: Shift x-values $+3$ units; shift y-values -2 units; shift z-values $+4$ units. Create the appropriate translation matrix:

$$T = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Let's translate the arbitrary point $(5, 6, 7)$, so apply T to the vector $\begin{pmatrix} 5 \\ 6 \\ 7 \\ 1 \end{pmatrix}$:

Then

$$T \begin{pmatrix} 5 \\ 6 \\ 7 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 6 \\ 7 \\ 1 \end{pmatrix} = \begin{pmatrix} 5+3 \\ 6-2 \\ 7+4 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \\ 11 \\ 1 \end{pmatrix} .$$

The resulting translated point is $(8, 4, 11)$.