Math 22A (Spring 2018)
Kouba
Exam 1

Your Name : KEY

Your EXAM ID Number ________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM ANOTHER STUDENT'S EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO HAVE ANOTHER STUDENT TAKE YOUR EXAM FOR YOU. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive LITTLE or NO credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

5. Make sure that you have 6 pages, including the cover page.

6. You have until 3 p.m. sharp to finish the exam. Failure to stop working when time is called may lead to a deduction in your exam score. Thank you for your cooperation.

1.) (11 pts.) Regular M & M's cost \$4/lb. and Almond M & M's cost \$8/lb. How many pounds of each type of M & M's should be mixed together in order to result in a mixture weighing 20 pounds and costing \$5/lb. ? Use any method to solve the problem.

Let x: lbs. reg. , y: lbs. alm. (lbs.)

$$\begin{cases} x+y=20 \\ 4x+8y=5(20)=100 \quad (\#) \end{cases} \rightarrow$$

$y=20-x \rightarrow$ (SUB) $\rightarrow 4x+8(20-x)=100$

$\rightarrow 4x+160-8x=100 \rightarrow 4x=60 \rightarrow$

$x=15$ lbs., $y=5$ lbs.

2.) Use Gauss-Jordan Reduction (reduced row echelon form) to solve each of the following systems of equations.

a.) (10 pts.) $\begin{cases} -3x+2y=7 \\ 4x+y=-2 \end{cases}$

$$\left[\begin{array}{cc|c} -3 & 2 & 7 \\ 4 & 1 & -2 \end{array}\right] \sim \left[\begin{array}{cc|c} -3 & 2 & 7 \\ 1 & 3 & 5 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 0 & 11 & 22 \\ 1 & 3 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 0 & 1 & 2 \\ 1 & 0 & -1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right]$$

$\rightarrow x=-1, \; y=2$

b.) (12 pts.) $\begin{cases} 2x + y - z = 2 \\ 3x - y + 2z = 1 \\ x - 2y + 3z = -1 \end{cases}$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 3 & -1 & 2 & 1 \\ 1 & -2 & 3 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 5 & -7 & 4 \\ 0 & 5 & -7 & 4 \\ 1 & -2 & 3 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 1 & -7/5 & 4/5 \\ 1 & -2 & 3 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 1 & -7/5 & 4/5 \\ 1 & 0 & 1/5 & 3/5 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1/5 & 3/5 \\ 0 & 1 & -7/5 & 4/5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\to \begin{cases} x + \frac{1}{5}z = 3/5 \\ y - \frac{7}{5}z = 4/5 \end{cases}$$ so let $\boxed{z = t}$ any # $\to$ $\boxed{x = 3/5 - 1/5\, t}$, $\boxed{y = 4/5 + 7/5\, t}$

3.) (12 pts.) TRUE or FALSE: If A and B are 2×2 matrices, then $AB = BA$. Prove that it's true or give a counterexample to show that it's false.

Let $A = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ then

$AB = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}$,

$BA = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$, so

$AB \neq BA$ $\boxed{\text{FALSE}}$

4.) Determine the inverse for each matrix.

a.) (10 pts.) $A = \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$

$$\left[\begin{array}{cc|cc} 5 & 7 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 1 & 1 & -2 \\ 2 & 3 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1 & 1 & 1 & -2 \\ 0 & 1 & -2 & 5 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 3 & -7 \\ 0 & 1 & -2 & 5 \end{array}\right]$$

$$\longrightarrow \quad A^{-1} = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix} \quad \text{OR}$$

$$\det A = (5)(3) - (7)(2) = 1 \rightarrow A^{-1} = \frac{1}{(1)} \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$$

b.) (12 pts.) $A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 2 & -2 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -2 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1/2 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 0 & 1/2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & -1 & 1/2 & 1/2 \end{array}\right] \rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 0 & 1/2 \\ -1 & 1/2 & 1/2 \end{bmatrix}$$

5.) (7 pts.) Consider the given matrices of different sizes. Some of the following matrix operations make sense and some do not make sense. After each part circle YES if it makes sense and circle NO if it does not make sense.

$$A = \begin{pmatrix} 2 \\ -3 \end{pmatrix},\ B = \begin{pmatrix} 4 & 1 \end{pmatrix},\ C = \begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix},\ D = \begin{pmatrix} 1 & 2 & 0 \\ -2 & 0 & 3 \end{pmatrix},\ E = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 0 & 4 \\ 0 & 5 & 0 \end{pmatrix}$$

a.) $A + B$ YES NO

b.) CD YES NO

c.) AB YES NO

d.) D^{-1} YES NO

e.) $E - D$ YES NO

f.) BA YES NO

g.) $AB + C$ YES NO

6.) (12 pts.) TRUE or FALSE: Assume that A and B are 2×2 matrices. Find two different matrices A and B, each with at least one nonzero number, so that $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, or explain why this is impossible.

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, then

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$ so

$\boxed{\text{TRUE}}$

7.) (14 pts.) For what values of constants a and b does the given augmented matrix lead to no solution, one solution, and infinity many solutions:

$$\left(\begin{array}{cc|c} 1 & a & a \\ 2 & 6 & b \end{array}\right)$$

$$\sim \left[\begin{array}{cc|c} 1 & a & a \\ 0 & 6-2a & b-2a \end{array}\right] \Rightarrow$$

I.) If $6-2a=0 \Rightarrow \boxed{a=3}$

case 1: If $b-2a=0 \Rightarrow b-6=0 \Rightarrow \boxed{b=6}$

$\Rightarrow$ last row: $[0\ 0 | 0]$, so infinitely many solutions

case 2: If $b-2a \neq 0 \Rightarrow b \neq 6 \Rightarrow$ last row is $[0\ 0 | *] \Rightarrow$ no solution

II.) If $6-2a \neq 0 \Rightarrow \boxed{a \neq 3} \Rightarrow$

$$\left[\begin{array}{cc|c} 1 & a & a \\ 0 & 6-2a & b-2a \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & a & a \\ 0 & 1 & c \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & a-c \\ 0 & 1 & c \end{array}\right]$$

$\Rightarrow$ unique solution

The following EXTRA CREDIT problem is OPTIONAL. It is worth 8 points.

1.) We say that a 2×2 matrix A is orthogonal if $AA^T = I$, where A^T is the transpose of A. Find three different examples of orthogonal matrices. At least one of the examples should contain no zeroes.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},$$

$$A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$