WH 3:

1a) Let

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ 0 & a_{22} & \cdots & a_{2m} \\ \vdots & & \ddots & \vdots \\ 0 & 0 & & a_{mm} \end{pmatrix}$$

We have:

$$\alpha_1 (a_{1m}, a_{2m}, \ldots, a_{mm})$$
$$+ \alpha_2 (a_{1m-1}, a_{2m-1}, \ldots, a_{m-1m-1}, 0)$$
$$+ \cdots$$
$$+ \alpha_m (a_{11}, 0, \ldots, 0)$$
$$= 0$$

Since $a_{mm} \neq 0$, then $\alpha_1 = 0$

$\therefore$ $a_{m-1m-1} \neq 0$, then $\alpha_2 = 0$

$\vdots$ $\vdots$

$a_{11} \neq 0$, then $\alpha_n = 0$

So $\alpha_k = 0 \ \forall \ 1 \leq k \leq m$

$\Rightarrow$ L.I. $\square$

1b) Yes, m independent columns have dim. m, i.e. span $\mathbb{R}^m$.

$\blacksquare$

2) $\dim W = \dim V = n \Rightarrow W = V$

We know that $W \subset V$, so it suffices to show $V \subset W$

Let $\psi = \{w_1, \dots, w_n\}$ be a base of W,

then ψ is L.I., since each $w_k \in W \subset V$

then ψ is a L.I. set w/ $\dim V = n$

elements and hence is a base.

Let $v \in V$, then $v = \sum_k \alpha_k w_k \in W$

so $V \subset W$ as desired.

$W = V \Rightarrow \dim W = \dim V = n$

Let $\psi = \{w_1, \dots, w_n\}$ be a base of W. Then ψ is a base of V since $W = V$. Then $\dim V = n = \dim W$. ∎