HW 4 Solutions

1a) First, we show $\det(A^T) = \det(A)$

$$\det(A) = \sum_{\pi \in S_m} \sigma(\pi)\, a_{1\pi_1} \cdots a_{m\pi_m}$$

$$\det(A^T) = \sum_{\pi \in S_m} \sigma(\pi)\, \underbrace{a_{\pi_1 1} \cdots a_{\pi_m m}}$$

This is just some permutation, write as

$a_{1p_1} a_{2p_2} \cdots a_{mp_m}$

$p = (p_1, \ldots, p_m) \in S_m$.

Must show $\sigma(\pi) = \sigma(p)$, i.e. π & p are both even or both odd.

In the language of the book (S-B), "p is the inverse permutation of π" and by 4.2.10, $\sigma(\pi) = \sigma(p)$.

$\square$

1b) $\det(A^{-1}BA)$

$$= \det(A^{-1})\det(B)\det(A)$$

$$= \det(A^{-1})\det(A)\det(B)$$

$$= \det(A^{-1}A)\det(B)$$

$$= \det(I)\det(B)$$

$$= \det(B)$$

$\square$

1c) This is not true

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\det(A) = \det(B) = 0$$

So $\quad \det(A) + \det(B) = 0$

but $\quad \det(A + B) = 1$ $\square$

1d)
$$\begin{aligned} 1 &= \det(I) \\ &= \det(AA^{-1}) \\ &= \det(A)\det(A^{-1}) \end{aligned}$$

$$\Rightarrow \quad \det(A^{-1}) = \frac{1}{\det A}$$ $\blacksquare$

2) Suppose m is odd.

Skew symmetric means $A = -A^T$

($\text{Symmetric: } A = A^T$)

then

$$\det(A) = \det(-A^T)$$
$$= (-1)^m \det(A^T)$$
$$= -\det(A) \quad (\text{since } m \text{ odd \& 1a})$$

$$\Rightarrow \det(A) = 0$$