MAT 22A SS1:

pw: easyquiz

Matricies

Data which admit represent. as matricies

→ linear systems of eqs.

$$
\begin{aligned}
3x + 4y + 2z &= 1 \\
2x + 8y &= 3 \\
7y + 5z &= 7
\end{aligned}
$$

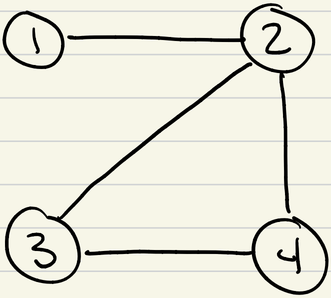
$$
\begin{pmatrix} 3 & 4 & 2 \\ 2 & 8 & 0 \\ 0 & 7 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}
$$

$$A \qquad \bar{x} \quad = \quad b$$

$$A\bar{x} = b$$

$$\boxed{\bar{x} = \frac{1}{A} b}$$

→ Graphs (computer science)



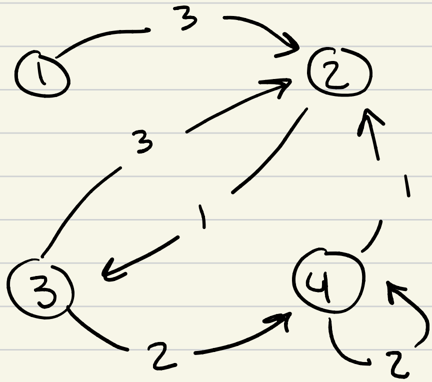
$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 0 & 1 & 1 & 0 \end{array}$$

✓ $$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

"symmetric matrix"

✓ $$\begin{bmatrix} 0 & 1 & \cdots \\ & \cdots & \end{bmatrix}$$

$$\cancel{\begin{vmatrix} \cdots \end{vmatrix}}$$



	1	2	3	4
1	0	3	0	0
2	0	0	1	0
3	0	3	0	2
4	0	1	0	2

$$\begin{pmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & 2 \end{pmatrix}$$

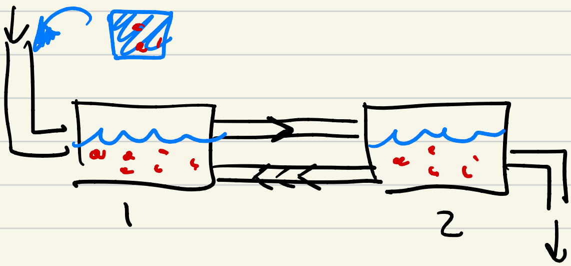
not symmetric

Ordinary Differential Eq.

→ Linear systems of ODEs

$$\frac{dy}{dx} = z$$

$$\frac{dz}{dx} = y$$



$$\frac{dy_1}{dt} = (i - u_1) y_1 + r_1 y_2$$

$$\frac{dy_2}{dt} = r_2 y_1 - u_2 y_2$$

$$\begin{pmatrix} i - u_1 & r_1 \\ r_2 & u_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_1' \\ y_2' \end{pmatrix}$$

↑ Derivs

$$A \frac{dy}{dt} = y$$

PW: graph

- Matrices are rectangular arrays of entries taken from a field $\mathbb{F}$.

$\boxed{\text{"field" or } \mathbb{F} \quad \text{think } \mathbb{R} \text{ or } \mathbb{C}}$

$$A = (a_{ij})_{m \times n} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

a_{ij} is the entry in ith row, jth column

$m \times n$ is m rows by n columns

$\mathbb{R}$ = real numbers

$\mathbb{C}$ = complex numbers $\quad \sqrt{-1} = i$

$a \in \mathbb{C} \Rightarrow a = b + ic, \quad b, c \in \mathbb{R}$

a is a element of $\mathbb{C}$

$$(a_{ij})_{m \times n} \qquad 1 \le i \le m \quad 1 \le j \le n$$

$$A = (a_{ij})_{m \times n}$$

" A is an $m \times n$ matrix over the field $\mathbb{F}$ "

Symbolically: $A \in \underbrace{\mathcal{M}_{m \times n}(\mathbb{F})}$

the set of $m \times n$ matricies with entries in $\mathbb{F}$

$$a_{ij} \in \mathbb{F}$$

ex: $A \in \mathcal{M}_{2 \times 2}(\mathbb{R})$

$$a_{11}, a_{12}, a_{21}, a_{22} \in \mathbb{R}$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

ex:

$$\begin{pmatrix} 1 & 2 & 4 \\ 7 & -3 & 8.\overline{13} \end{pmatrix} \in \mathcal{M}_{2\times 3}(\mathbb{R})$$

$$\begin{pmatrix} 4 + 3i \\ 1 + 9i \end{pmatrix} \in \mathcal{M}_{2\times 1}(\mathbb{C})$$

- "vectors": commonly it means

$$v \in \mathcal{M}_{1\times n}(\mathbb{R}) \quad \text{or} \quad v \in \mathcal{M}_{m\times 1}(\mathbb{R})$$

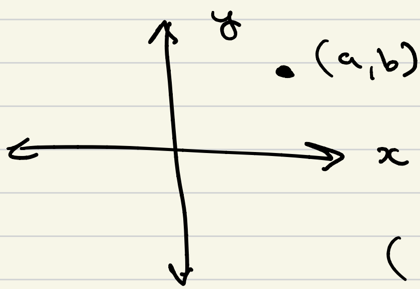
→ $\mathcal{M}_{1\times n}(\mathbb{R})$ row vector

$$\begin{pmatrix} 1 & 2 & 9 & 6 \end{pmatrix} \in \mathcal{M}_{1\times 4}(\mathbb{R})$$

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \in \mathcal{M}_{1\times 2}(\mathbb{R})$$

$$\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \end{pmatrix} \in \mathcal{M}_{1\times n}(\mathbb{R})$$

$\mathbb{R}^2$



$$(a,\ b)$$
$$\updownarrow \quad \updownarrow$$
$$\begin{pmatrix} a & b \end{pmatrix}$$

- "Isomorphism"

Definition: Two sets A & B are <u>isomorphic</u> if there is a function F from A to B such that for every $a \in A$, there is a unique $b \in B$ such that $F(a) = b$

And for every $b \in B$, there is a unique $a \in A$ such that $F(a) = b$ or $a = F^{-1}(a)$.

We write $A \cong B$ and say A is isomorphic to B.

Claim: $\mathbb{R}^n \cong \mathcal{M}_{1 \times n}(\mathbb{R}) \cong \mathcal{M}_{n \times 1}(\mathbb{R})$

What does $a \in \mathbb{R}^n$ look like?

$a \in \mathbb{R}^n$ means

$a = (a_1, a_2, \cdots, a_n)$, $a_i \in \mathbb{R}$

What does $b \in M_{1\times n}(\mathbb{R})$ look like?

$$b = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix}$$

$$a = (a_1, a_2, \cdots, a_n) \in \mathbb{R}^n$$

$$\updownarrow \quad \updownarrow \quad \quad \updownarrow$$

$$\bar{a} = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} \in M_{1\times n}(\mathbb{R})$$

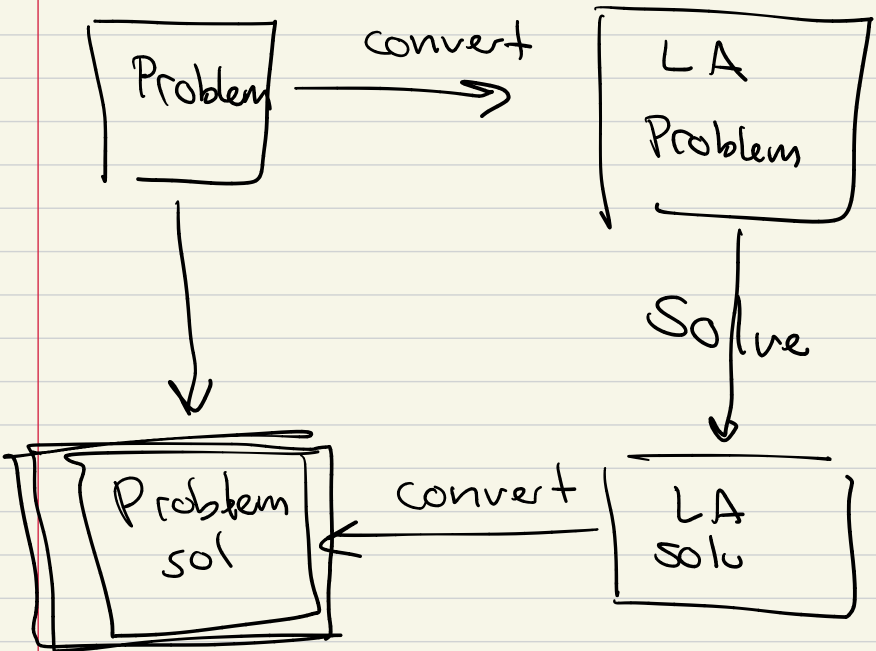
$$\boxed{F((a_1, a_2, \cdots, a_n)) = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}}$$

$$\Rightarrow \mathbb{R}^n \cong M_{1\times n}(\mathbb{R}).$$

$$\bar{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \in M_{n\times 1}(\mathbb{R})$$

Try this one on your own.

$$\mathbb{R}^n \cong M_{n\times 1}(\mathbb{R}).$$



ex problem: Integration in 3 space

- Preview: linear systems of Eqs.

Big idea: convert to matrix eqs. and solve those

$$
\begin{aligned}
x + y + z &= 1 \\
2x + 2y + z &= 1 \\
x + 2y + 3z &= 3
\end{aligned}
\qquad (*)
$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix}, \quad \bar{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in M_{3\times1}(\mathbb{R})$$

$$b = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \in M_{3\times1}(\mathbb{R})$$

We write $(*)$ as

$$A\bar{x} = b$$

If $A \in \mathbb{R}$, $\bar{x} \in \mathbb{R}$, $b \in \mathbb{R}$, solution is

$$\bar{x} = \frac{1}{A} b, \quad A \neq 0$$

$$= A^{-1} b, \quad A \in \mathbb{R} \Rightarrow A^{-1} = \frac{1}{A}$$

$\bar{x} = A^{-1} b$

we need to

- Define A^{-1}
- Define multiplication between A & $\bar{x}$ and A^{-1} & b

$\underbrace{\bar{x} = A^{-1} b}$ is the solution

this is "easy" to compute

- Algebra w/ matricies
  - → Two simple concepts:

addition & <u>scalar</u> multiplication

$A, B \in \mathcal{M}_{m \times n}(\mathbb{F})$

$A + B$

$A = (a_{ij})_{m \times n}, \quad B = (b_{ij})_{m \times n}$

$$\underbrace{A + B}_{\in \mathcal{M}_{m \times n}(\mathbb{F})} := (a_{ij} + b_{ij})_{m \times n}$$

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & -3 \\ 9 & 1 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 5 & -1 \\ 13 & 3 \end{pmatrix}$$

- <u>Scalar</u> multiplication

Scalar ~ $\mathbb{R}$ or $\mathbb{C}$

$$\alpha \in \mathbb{R} \quad , \quad A \in \mathcal{M}_{m \times n}(\mathbb{R})$$

$$\alpha A := (\alpha a_{ij})_{m \times n}$$

Ex :

$$A = \begin{pmatrix} 3 & 2 \\ 4 & -3 \\ -9 & 7 \end{pmatrix}$$

$$2A = \begin{pmatrix} 6 & 4 \\ 8 & -6 \\ -18 & 14 \end{pmatrix}$$