MAT 022A Linear Algebra | Homework 1 | Due June 28, 11:59pm

> Answer all questions in a clear and concise manner. Show all work. Submit one page per problem on Gradescope, properly select the subproblems and indicate if the solution requires multiple pages.
> You may consult the internet, book, and peers. Cite any sources other than the book that you used. Citing Stack Exchange/Stack Overflow and/or Chegg is -.5 points per citation (subject to caveats discussed in lecture). If you are suspected of using an online resource without citation you will receive zero points for that problem.
> If you work with peers, include the names of all in your group and do your own write-up **independently**, you should understand the solutions you are including in your write up. Copying on problems will result in zero points on that section.

1. In general, matrices do not commute. However it is always the case that $AA^{-1} = I = A^{-1}A$. There is a special subset of $Z \subset \mathcal{M}_{m\times m}(\boldsymbol{R})$ called the **center** of $\mathcal{M}_{m\times m}(\boldsymbol{R})$ which consists of precisely the matrices B for which $AB = BA$ for every $A \in \mathcal{M}_{m\times m}(\boldsymbol{R})$. In other words, elements in the center commute via multiplication with all matrices. For example, $I \in Z$ since $AI = A = IA$ for all $A \in \mathcal{M}_{m\times m}(\boldsymbol{R})$.
   (a) (2 points) Find the general form of a matrix $B \in Z \subset \mathcal{M}_{2\times 2}(\boldsymbol{R})$. In other words, describe the center of the set of 2×2 real valued matrices.
   (b) (3 points) Prove that the center of $\mathcal{M}_{m\times m}(\boldsymbol{R})$ is a vector space under matrix addition (with scalars in $\boldsymbol{R}$).
2. A matrix A is nilpotent if $A^2 = AA = 0$.
   (a) (1 point) Describe the general form of the nilpotent matrices of $\mathcal{M}_{2\times 2}(\boldsymbol{R})$.
   (b) (1 point) Find a matrix $A \in \mathcal{M}_{2\times 2}(\boldsymbol{R})$ such that $A^2 \neq 0$, but $A^3 = 0$. Carry out your computations.
   (c) (1 points) Let V consist of matrices of the form

$$\begin{pmatrix} 1 & 0 & \alpha \\ 0 & \beta & \alpha \\ 1 & 1 & \beta \end{pmatrix}$$

   where α, β are arbitrary real numbers. Find all matrices $B \in \mathcal{M}_{3\times 3}(\boldsymbol{R})$ such that for any $A \in V$, $BA = 0$. We call the set of all B the **annihilator** of V.
3. (2 points) Prove that the space of continuous functions $C^0(\boldsymbol{R})$ is a vector space under standard addition (with scalars in $\boldsymbol{R}$).