MAT 022A Linear Algebra | Homework 3 | Due July 12, 11:59pm

> Answer all questions in a clear and concise manner. Show all work. Submit one page per problem on Gradescope, properly select the subproblems and indicate if the solution requires multiple pages.
> You may consult the internet, book, and peers. Cite any sources other than the book that you used. Citing Stack Exchange/Stack Overflow and/or Chegg is -.5 points per citation (subject to caveats discussed in lecture). If you are suspected of using an online resource without citation you will receive zero points for that problem.
> If you work with peers, include the names of all in your group and do your own write-up **independently**, you should understand the solutions you are including in your write up. Copying on problems will result in zero points on that section.

1. A square matrix $A \in \mathcal{M}_{m\times m}(\boldsymbol{R})$ is called **upper triangular** if $a_{ij} = 0$ whenever $j > i$. For example

$$\begin{pmatrix} 1 & 8 & -7 \\ 0 & 4 & 6 \\ 0 & 0 & -3 \end{pmatrix} \in \mathcal{M}_{3\times 3}(\boldsymbol{R})$$

   is upper triangular.

   Let $A \in \mathcal{M}_{m\times m}(\boldsymbol{R})$ and suppose that for all $1 \leqslant i \leqslant m$, $a_{ii} \neq 0$.

   (a) (2 points) Prove that the columns, regarded as column vectors, are linearly independent.

   (b) (2 points) Do the columns span $\boldsymbol{R}^m$?

2. (3 points) Let V be an n-dimensional vector space, and $W \subset V$ a subspace of V. Prove that $\dim W = n$ if and only if $W = V$; that is every element of V is also in W.