

Answer all questions in a clear and concise manner. Show all work. Submit one page per problem on Gradescope, properly select the subproblems and indicate if the solution requires multiple pages.

You may consult the internet, book, and peers. Cite any sources other than the book that you used. Citing Stack Exchange/Stack Overflow and/or Chegg is -.5 points per citation (subject to caveats discussed in lecture). If you are suspected of using an online resource without citation you will receive zero points for that problem.

If you work with peers, include the names of all in your group and do your own write-up **independently**, you should understand the solutions you are including in your write up. Copying on problems will result in zero points on that section.

1. Let $A, B \in \mathcal{M}_{d \times d}(\mathbf{R})$, and let A be invertible. Prove or disprove the following
 - (a) (1 point) $\det(A^T A) = \det(A^2)$.
 - (b) (1 point) $\det(A^{-1} B A) = \det B$.
 - (c) (1 point) $\det(\sum_{k=1}^N A_k) = \sum_{k=1}^N \det A_k$, for $A_k \in \mathcal{M}_{d \times d}(\mathbf{R})$, $k = 1, 2, \dots, N$.
 - (d) (1 point) $\det(A^{-1}) = \frac{1}{\det(A)}$.
2. (2 points) A matrix $A \in \mathcal{M}_{m \times m}(\mathbf{R})$ is skew symmetric if $a_{ij} = -a_{ji}$ for all i, j . Prove that if m is odd, $\det A = 0$.