1. Let $A, B \in \mathcal{M}_{d \times d}(\mathbf{R})$, and let A be invertible. Prove or disprove the following

   (a) (1 point) $\det(A^T A) = \det(A^2)$.

   (b) (1 point) $\det(A^{-1}BA) = \det B$.

   (c) (1 point) $\det(\sum_{k=1}^{N} A_k) = \sum_{k=1}^{N} \det A_k$, for $A_k \in \mathcal{M}_{d \times d}(\mathbf{R})$, $k = 1, 2, \cdots, N$.

   (d) (1 point) $\det(A^{-1}) = \frac{1}{\det(A)}$.

2. (2 points) A matrix $A \in \mathcal{M}_{m \times m}(\mathbf{R})$ is skew symmetric if $a_{ij} = -a_{ji}$ for all i, j. Prove that if m is odd, $\det A = 0$.