MAT 022A Linear Algebra Homework 5 (Last Written Homework) Due July 29, 11:59pm

Answer all questions in a clear and concise manner. Show all work. Submit one page per problem on Gradescope, properly select the subproblems and indicate if the solution requires multiple pages.
You may consult the internet, book, and peers. Cite any sources other than the book that you used. Citing Stack Exchange/Stack Overflow and/or Chegg is -.5 points per citation (subject to caveats discussed in lecture). If you are suspected of using an online resource without citation you will receive zero points for that problem.
If you work with peers, include the names of all in your group and do your own write-up **independently**, you should understand the solutions you are including in your write up. Copying on problems will result in zero points on that section.

Assume all matricies are $\mathcal{M}_{m\times m}(\boldsymbol{R})$ unless otherwise specified.

1. (1 point) Prove or disprove that the eigenvalues of A and A^T are the same.
2. (2 points) Let A be a matrix with m distinct, non-zero, eigenvalues. Prove that the eigenvectors of A are linearly independent and span $\boldsymbol{R}^m$. Note that this means (in this case) that the eigenvectors are distinct and form a base of the space.
3. (1 point) Given that λ is an eigenvalue of A associated with v, prove that v is an eigenvector of A^k and that λ^k is it's associated eigenvalue.
4. (2 points) Let A be a matrix with eigenvalues λ_k, $1 \leqslant k \leqslant m$. Show that $\det A = \lambda_1\lambda_2\cdots\lambda_m$.
5. (1 point) Prove that a matrix A is singular if and only if it has a zero eigenvalue (hint: use the previous problem).
6. Recall that we may define the exponential function in terms of it's Taylor series expansion

   $$e^x = \sum_{k=0}^{\infty} \frac{1}{k!}x^k$$

   Similarly, we define the **matrix exponential** in terms of the following series

   $$e^A = \sum_{k=0}^{\infty} \frac{1}{k!}A^k$$

   Where A^k is matrix multiplication of A with itself k times.

   Compute the matrix exponential for the following matricies

   (a) (1 point) I_2.

   (b) (1 point) $\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$.

   (c) (1 point) Any 2×2 nilpotent matrix (recall homework 2)

   (d) (1 point) Will every 2×2 matrix have a convergent matrix exponential? Why or why not? (Recall your Taylor series from 21C).