Answer all questions in a clear and concise manner. Show all work. Submit one page per problem on Gradescope, properly select the subproblems and indicate if the solution requires multiple pages.
You may consult the internet, book, and peers. Cite any sources other than the book that you used. Citing Stack Exchange/Stack Overflow and/or Chegg is -.5 points per citation (subject to caveats discussed in lecture). If you are suspected of using an online resource without citation you will receive zero points for that problem.
If you work with peers, include the names of all in your group and do your own write-up **independently**, you should understand the solutions you are including in your write up. Copying on problems will result in zero points on that section.

1. Prove that if A is invertible and symmetric, i.e. $a_{ij} = a_{ji}$, prove that A^{-1} is symmetric.
2. Show that for $A \in \mathcal{M}_{m\times m}$, $\alpha \in \boldsymbol{R}$, $\det(\alpha A) = \alpha^m \det A$.
3. A matrix A is called **orthogonal** if A is invertible and $A^{-1} = A^T$.
   (a) Prove that $\det A$ is either 1 or -1.
   (b) The **special linear group**, SL_m consists of the orthogonal matrices which have $\det A = 1$. Describe the elements of SL_2.
   (c) (Bonus: Don't spend too much time on this one) Come up with a criteria for membership in SL_m based on the eigenvalues of a matrix.
4. Let $A, B \in \mathcal{M}_{m\times m}$. If $AB = -BA$ and m is odd, prove that A and B cannot both be invertible.
5. Let $A, B \in \mathcal{M}_{m\times m}(\boldsymbol{R})$ and let v be an eigenvector of A with eigenvalue λ, and v be an eigenvector of B with eigenvalue μ.
   (a) Show that v is an eigenvector of AB. What is the corresponding eigenvalue?
   (b) Show that v is an eigenvector of $A + B$. What is the corresponding eigenvalue?
6. We have seen that A and A^T have the same eigenvalues. Do they have the same eigenvectors? Why or why not?
7. Prove that the eigenvalues of an upper triangular matrix are the diagonal entries.
8. (From Differential Equations and Linear Algebra by Goode and Annin) We say that a matrix B is a square root of A if $B^2 = A$.
   (a) Show that the if $D = \mathrm{diag}(\lambda_1, \cdots, \lambda_m)$, then $\sqrt{D} = \mathrm{diag}(\sqrt{\lambda_1}, \cdots, \sqrt{\lambda_m})$.
   (b) Show that if A is nondefective and $P^{-1}AP = D$ for P invertible, then $P\sqrt{D}P^{-1}$ is a square root of A.
   (c) Find a square root for
   $$A = \begin{pmatrix} 6 & -2 \\ -3 & 7 \end{pmatrix}$$
   (d) Is the square root of a matrix unique?
9. Let J be a Jordan block and prove that the Jordan canonical form of J^T is J.
10. Prove that A and A^T have the same Jordan canonical form.
11. (From HW 2) Prove that there is no matrix A such that $A^2 \neq 0$ but $A^3 = 0$.