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ERRATUM: NONLINEAR OBSERVER DESIGN IN THE SIEGEL DOMAIN*

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Abstract. There is an error in the proof of the main result of our paper [*SIAM J. Control Optim.*, 41 (2002), pp. 932–953]. An additional assumption is needed for the main result to hold. In this erratum, we supply a corrected version of the main result.

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In our paper [1], we considered the problem of transforming the real analytic system

$$\dot{x} = f(x) = Fx + \cdots, \tag{0.1}$$

$$y = h(x) = Hx + \cdots \tag{0.2}$$

by a local, analytic change of coordinates

$$z = \theta(x)$$

and an analytic output injection

$$\beta(y)$$

into the system

$$\dot{z} = Az + \beta(y).$$

As shown by Kazantzis and Kravaris [2], this question is of interest because the latter system admits an observer

$$\dot{\hat{z}} = A\hat{z} + \beta(y)$$

with linear error dynamics

$$\dot{\tilde{z}} = A\tilde{z},$$

where $\tilde{z} = z - \hat{z}$. Such a $\theta(x)$ and $\beta(y)$ must satisfy the PDE

$$\frac{\partial \theta}{\partial x}(x) f(x) = A\theta(x) + \beta(h(x)). \tag{0.3}$$

Using the Lyapunov auxiliary theorem, Kazantzis and Kravaris showed, given an analytic β, that this PDE admits a unique solution if all the eigenvalues of F lie in the same half plane, either the left or the right, and the eigenvalues of A are not resonant with those of F.

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In our paper, we claimed (Theorem 2) the existence of solutions to this PDE under considerably weaker hypothesis on the spectrum of F, but there is an error in our proof. An additional assumption is required: that the eigenvalues of the linear part of the system are of type (C, ν) with respect to (w.r.t.) themselves. The correct statement of the main result is as follows.

MAIN THEOREM. *Assume that* $f : \mathbf{R}^n \to \mathbf{R}^n, h : \mathbf{R}^n \to \mathbf{R}^p$, *and* $\beta : \mathbf{R}^p \to \mathbf{R}^n$ *are analytic vector fields with* $f(0) = 0,\ h(0) = 0,\ \beta(0) = 0$, *and* $F = \frac{\partial f}{\partial x}(0),\ H = \frac{\partial h}{\partial x}(0),\ B = \frac{\partial \beta}{\partial y}(0)$. *Suppose*

1. *there exists an invertible* $n \times n$ *matrix* T *so that* $TF = AT - BH$,
2. *there exists a* $C > 0, \nu > 0$ *such that all the eigenvalues of* A *are of type* (C, ν) *w.r.t.* $\sigma(F)$,
3. *there exists a* $C > 0, \nu > 0$ *such that all the eigenvalues of* F *are of type* (C, ν) *w.r.t.* $\sigma(F)$.

Then there exists a unique analytic solution $z = \theta(x)$ *to the PDE* (0.3) *locally around* $x = 0$. *Moreover* $\frac{\partial \theta}{\partial x}(0) = T$, *so* θ *is a local diffeomorphism.*

Proof. Because the eigenvalues of F are of type (C, ν) w.r.t. themselves, it follows from Siegel's theorem [3] that there exists an analytic local change of coordinates which linearizes the dynamics of the system (0.1). Therefore without loss of generality we may assume that the system is of the form

$$\dot{x} = Fx, \tag{0.4}$$

$$y = h(x). \tag{0.5}$$

Then the PDE (0.3) becomes

$$\frac{\partial \theta}{\partial x}(x) F x = A\theta(x) + \beta(h(x)), \tag{0.6}$$

which has a unique local solution by Theorem 1 of [1]. □

Remarks. With the additional assumption, our result is no longer a generalization of that of Kazantzis and Kravaris [2]. The set of complex vectors $\lambda = (\lambda_1, \ldots, \lambda_n) \in \mathbb{C}^n$ which are not of type (C, ν) w.r.t. themselves for any C is a set of measure zero if ν is large enough [3]. Therefore the additional assumption is satisfied by almost all systems.

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