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Nonlinear Observer Design for State and Disturbance Estimation

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Abstract—**A new systematic framework for nonlinear observer design that allows the concurrent estimation of the process state variables, together with key unknown process or sensor disturbances is proposed. The nonlinear observer design problem is addressed within a similar methodological framework as the one introduced in [8,11] for state estimation purposes only. From a mathematical standpoint, the problem under consideration is addressed through a system of first-order singular PDEs for which a rather general set of solvability conditions is derived. A nonlinear observer can then be designed with a state-dependent gain that can be computed from the solution of the system of singular PDEs. Under the aforementioned conditions, both state and disturbance estimation errors converge to zero with assignable rates. The convergence properties of the proposed nonlinear observer are tested through simulation studies in an illustrative example involving a biological reactor.**

I. Introduction

TECHNICAL limitations and/or prohibitively high cost associated with current sensor technology and measurement procedures entail the non-availability of all process state variables for direct on-line measurements. Furthermore, key process parameters frequently represent unknown or poorly known time-varying disturbances, that are particularly encountered in areas such as catalytic reaction engineering, bioprocess engineering and environmental engineering [2]. The operation of sensing devices is also subject to external disturbances, involving for example a sudden or gradual decalibration of the instrument. Therefore, there is an essential need for an accurate estimation of the unmeasurable process state variables together with key process or sensor disturbances, especially when they are used in the synthesis of model-based controllers or for direct process monitoring purposes [14]. For the combined state and disturbance estimation task, an observer can be employed. In the case of linear systems, both the well-known Kalman filter and its deterministic analogue realized by Luenberger's observer [2], offer a comprehensive solution to the problem under consideration. However, most chemical and physical processes are inherently nonlinear and nonlinear observers need to be designed that are capable of directly coping with the process nonlinearities. The nonlinear observer design problem is much more challenging and has received a considerable amount of attention in the literature leading to various approaches with different methodological characteristics [1,3-15]. The present research work aims at the development of a new systematic and practical framework for nonlinear observer design that allows the concurrent estimation of the state variables, together with key unknown process or sensor disturbances. In particular, the nonlinear observer design problem is formulated and addressed within a similar methodological framework as the one introduced in [8,11] for state estimation purposes only, and from a mathematical standpoint, via a new system of first-order singular PDEs for which a rather general set of necessary and sufficient conditions for solvability is derived. A nonlinear observer can then be designed that possesses a state-dependent gain computed from the solution of the above system of singular PDEs. Under the above conditions, it can be proven that both state and disturbance estimation errors converge to zero with assignable rates. Finally, the performance of the proposed observer is evaluated in an illustrative bioreactor application through simulation studies.

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II. Problem Formulation

Consider a dynamic system

$$\begin{aligned} \dot{x} &= f(x,w) \\ y &= h(x,w) \end{aligned} \tag{1}$$

that represents the dynamics of a process, where x is the process state vector, y is the vector of measurements and w is the vector of unmeasurable process or sensor disturbances. The dynamics of the disturbances is governed by the exosystem

$$\dot{w} = s(w) \tag{2}$$

The problem of *state and disturbance estimation* becomes a pure state estimation problem when one considers the extended system:

$$\begin{aligned} \dot{x} &= f(x,w) \\ \dot{w} &= s(w) \\ y &= h(x,w) \end{aligned} \tag{3}$$

where $\begin{bmatrix} x \\ w \end{bmatrix}$ is the extended system's state vector, which must be estimated via an appropriately designed observer.

A special case of the above problem is the *state and sensing error estimation problem*, where the disturbances affect the sensing devices only, and in an additive way:

$$\begin{aligned} \dot{x} &= f(x) \\ \dot{w} &= s(w) \\ y &= h(x) + q(w) \end{aligned} \tag{4}$$

For the state and sensing error estimation problem, special results will be presented, which significantly simplify the design of the observer.

The approach that will be taken in this work is the *observer error linearization* approach ([8], [11]), where the observer is designed so that, after coordinate transformation, the error dynamics is linear and with prescribed eigenvalues.

Generally speaking, the degree of difficulty of the observer design problem depends on the nature of the eigenvalues of the linearization of the extended system, in particular whether their convex hull includes the origin (spectrum is in the Siegel domain) or does not include the origin (spectrum is in the Poincaré domain).

The following definitions will be needed for the rest of the paper:

- Given a set of eignevalues $\lambda_1,\ldots,\lambda_n$, a complex number μ is said to be *nonresonant* with this set of eigenvalues if it is not related with them through any relation of the form $\mu = \sum_{i=1}^{n} m_i \lambda_i$, where $m_1,\ldots,m_n$ are nonnegative integers not all zero.
- Given a set of eigenvalues $\lambda_1,\ldots,\lambda_n$, a complex number μ is said to be *of type* (C,ν) with respect to this set of eigenvalues if there exist constants $C>0$ and $\nu>0$ such that $\left|\mu - \sum_{i=1}^{n} m_i\lambda_i\right| \geq \dfrac{C}{\left(\sum_{i=1}^{n} m_i\right)^{\nu}}$ for any nonnegative integers $m_1,\ldots,m_n$ that are not all zero.

As will be seen in the next section, nonresonance conditions will arise if the spectrum of the linearization of (3) is in the Poincaré domain, otherwise (C,ν) conditions will arise.

III. Nonlinear Observer Design

Consider the system (3)

$$\begin{aligned} \dot{x} &= f(x,w) \\ \dot{w} &= s(w) \\ y &= h(x,w) \end{aligned}$$

where $f: R^n \times R^\ell \to R^n$, $s: R^\ell \to R^\ell$, $h: R^n \times R^\ell \to R^p$ are real analytic functions, with $f(0,0)=0$, $s(0)=0$, $h(0,0)=0$. A nonlinear observer for (3) can be designed following the methodology introduced in [8,11]. A local diffeomorphism $z=\theta(x,w)$ is sought that maps the system (3) into

$$\dot{z} = Az + \beta(y) \tag{5}$$

where A is a $(n+\ell)\times(n+\ell)$ matrix and $\beta: R^p \to R^{n+\ell}$ is a real analytic function with $\beta(0)=0$. As long as such a transformation can be found, (5) can be used as observer dynamics and the inverse transformation can be used to reconstruct the system states:

$$\begin{aligned} \dot{\hat{z}} &= A\hat{z} + \beta(y) \\ \begin{bmatrix} \hat{x} \\ \hat{w} \end{bmatrix} &= \theta^{-1}(\hat{z}) \end{aligned} \tag{6}$$

It turns out that the unknown transformation map θ must satisfy the following system of singular PDEs [8,11]:

$$\frac{\partial\theta}{\partial x}(x,w)f(x,w) + \frac{\partial\theta}{\partial w}(x,w)\,s(w) = A\theta(x,w) + \beta(h(x,w)) \tag{7}$$

Therefore, the problem of interest reduces to the study of the PDEs (7) and the properties of the solution. The following Propositions are direct consequences of the results in [8] and [11,12] respectively.

<u>*Proposition 1:*</u> *Let* $f: R^n \times R^\ell \to R^n$, $s: R^\ell \to R^\ell$, $h: R^n \times R^\ell \to R^p$ *and* $\beta: R^p \times R^{n+\ell}$ *be real analytic vector functions with* $f(0,0)=0$, $s(0)=0$, $h(0,0)=0$, $\beta(0)=0$ *and* $F=\dfrac{\partial f}{\partial x}(0,0)$, $P=\dfrac{\partial f}{\partial w}(0,0)$, $S=\dfrac{\partial s}{\partial w}(0)$, $H=\dfrac{\partial h}{\partial x}(0,0)$, $Q=\dfrac{\partial h}{\partial w}(0,0)$, $B=\dfrac{\partial \beta}{\partial y}(0)$. *Denote by* $\sigma(F)$ *and* $\sigma(S)$ *the spectra of S and F respectively.*

Suppose:

1. There exists an invertible $(n+\ell)\times(n+\ell)$ *matrix* T *such that* $T\begin{bmatrix} F & P \\ 0 & S \end{bmatrix} = AT + B\begin{bmatrix} H & Q \end{bmatrix}$

2. All the eigenvalues of A are non-resonant with $\sigma(F)\cup\sigma(S)$.

3. 0 does not lie in the convex hull of $\sigma(F)\cup\sigma(S)$.

Then there exists a unique analytic solution $z=\theta(x,w)$ *to the PDE (7) locally around (x,w)=(0,0). The solution has the property that* $\left[\frac{\partial\theta}{\partial x}(0,0) \quad \frac{\partial\theta}{\partial w}(0,0)\right]=T$ *and so,* θ *is a local diffeomorphism.*

Proposition 2: *Under the notations of Proposition 1, suppose:*

1. There exists an invertible $(n+\ell)\times(n+\ell)$ *matrix T such that* $T\begin{bmatrix} F & P \\ 0 & S \end{bmatrix}=AT+B\begin{bmatrix} H & Q \end{bmatrix}$

2. There exist C > 0, $\nu>0$ *such that all the eigenvalues of A are of type* $(C,\ \nu)$ *with respect to* $\sigma(F)\cup\sigma(S)$.

3. There exist C > 0, $\nu>0$ *such that all the eigenvalues of F and S are of type* $(C,\ \nu)$ *with respect to* $\sigma(F)\cup\sigma(S)$.

Then there exists a unique analytic solution $z=\theta(x,w)$ *to the PDE (7) locally around (x,w)=(0,0). The solution has the property that* $\left[\frac{\partial\theta}{\partial x}(0,0) \quad \frac{\partial\theta}{\partial w}(0,0)\right]=T$ *and so,* θ *is a local diffeomorphism.*

Remark 1: Assumptions 1 and 2 of either Proposition imply that $\left(\begin{bmatrix} H & Q \end{bmatrix}, \begin{bmatrix} F & P \\ 0 & S \end{bmatrix}\right)$ is an observable pair. On the other hand, if $\left(\begin{bmatrix} H & Q \end{bmatrix}, \begin{bmatrix} F & P \\ 0 & S \end{bmatrix}\right)$ is an observable pair, it is always possible to find matrices *A, B, T* which satisfy the matrix equation of Assumption 1, with T invertible and *A* having prescribed eigenvalues.

Remark 2: Under the assumption that $\sigma(F)$ and $\sigma(S)$ are disjoint sets, it is possible to show that the pair of composite matrices $\left(\begin{bmatrix} H & Q \end{bmatrix}, \begin{bmatrix} F & P \\ 0 & S \end{bmatrix}\right)$ is observable if and only if the following conditions hold:

a) (H , F) is an observable pair

b) ($HR+Q$, S) is an observable pair, where R is the solution of $RS-FR=P$.

Condition b) implies that no eigenvalue of S is a transmission zero of (F, P, H, Q).

It should be noted that observer (6) can be expressed in the original coordinates via the inverse transformation θ^{-1}, so that $\hat{x}$ and $\hat{w}$ explicitly represent the observer states:

$$\begin{aligned} \dot{\hat{x}} &= f(\hat{x},\hat{w})+L_x(\hat{x},\hat{w})\{\beta(y)-\beta(h(\hat{x},\hat{w}))\} \\ \dot{\hat{w}} &= s(\hat{w})+L_w(\hat{x},\hat{w})\{\beta(y)-\beta(h(\hat{x},\hat{w}))\} \end{aligned} \tag{8}$$

where $\begin{bmatrix} L_x(\hat{x},\hat{w}) \\ L_w(\hat{x},\hat{w}) \end{bmatrix}=\left[\frac{\partial\theta}{\partial x}(\hat{x},\hat{w}) \quad \frac{\partial\theta}{\partial w}(\hat{x},\hat{w})\right]^{-1}$.

Finally, it should be noted that the estimation error in the transformed coordinates follows linear dynamics, governed by the arbitrarily selected matrix A (design parameter):

$$\frac{d}{dt}\left[\theta(x,w)-\theta(\hat{x},\hat{w})\right]=A\left[\theta(x,w)-\theta(\hat{x},\hat{w})\right].$$

IV. Nonlinear Observer Design for State and Sensing Error Estimation

Consider now the special case where the disturbances affect the sensing devices only, and in an additive way:

$$\begin{aligned} \dot{x} &= f(x) \\ \dot{w} &= s(w) \\ y &= h(x)+q(w) \end{aligned} \tag{9}$$

Also, suppose that for the design of the observer, linear output injection $\beta(y)=By$ is used, where B is a $(n+\ell)\times\rho$ matrix. Then, the system of PDEs (7) becomes:

$$\begin{aligned} &\frac{\partial\theta}{\partial x}(x,w)f(x)+\frac{\partial\theta}{\partial w}(x,w)s(w)= \\ &A\theta(x,w)+\mathrm{B}(h(x)+q(w)) \end{aligned} \tag{10}$$

The solution of (10) can be expressed as:

$$\theta(x,w)=\psi(x)+\omega(w) \tag{11}$$

where ψ and ω satisfy the following system of PDE's:

$$\frac{\partial\psi}{\partial x}(x)f(x)=A\psi(x)+Bh(x) \tag{12}$$

$$\frac{\partial\omega}{\partial w}(w)s(w)=A\omega(w)+Bq(w) \tag{13}$$

In this way, the system of PDEs for θ (10) is broken into two decoupled sub-systems of PDE's of smaller dimension, and therefore, the computational effort is significantly reduced. PDE (12) is exactly the PDE for the observer design for the disturbance-free part of the system, whereas (13) is the corresponding observer PDE for the disturbance dynamics.

For $\theta(x,w)$ of the form (11), with ψ and ω being solutions of (12) and (13), the observer (8) takes the form:

$$\begin{aligned} \dot{\hat{x}} &= f(\hat{x})+L_x(\hat{x},\hat{w})(y-h(\hat{x})-q(\hat{w})) \\ \dot{\hat{w}} &= s(\hat{w})+L_w(\hat{x},\hat{w})(y-h(\hat{x})-q(\hat{w})) \end{aligned} \tag{14}$$

where the corresponding gains are given by the following expressions:

$$\begin{bmatrix} L_x(\hat{x},\hat{w}) \\ L_w(\hat{x},\hat{w}) \end{bmatrix}=\left[\frac{\partial\psi}{\partial x}(x) \quad \frac{\partial\omega}{\partial w}(w)\right]^{-1}B \tag{15}$$

It should be noted that in most engineering applications, there are two further simplifications:

i) the disturbances w are considered to be prototype disturbances (e.g. steps, ramps, sine waves, etc.) that follow linear dynamics, which means $s(\cdot)$ and $q(\cdot)$ are linear functions:

$$\begin{aligned} s(w) &= Sw \\ q(w) &= Qw \end{aligned} \tag{16}$$

where S and Q are matrices of appropriate dimensions.

Then the solution to the PDE (13) is also a linear function:

$$\omega(w) = \Omega w \tag{17}$$

where Ω is the solution of the matrix equation $\Omega S - A\Omega = BQ$.

ii) The dynamics of the process states $\dot{x} = f(x)$ is hyperbolically stable, which means that the eigenvalues of its linearization are in the Poincaré domain. Then the results in [8] lead to the following:

<u>Proposition 3</u>: *Let $f: R^n \to R^n$ and $h: R^n \to R^\rho$ be real analytic vector functions with $f(0)=0$, $h(0)=0$ and $F = \frac{\partial f}{\partial x}(0)$, $H = \frac{\partial h}{\partial x}(0)$.*

Suppose:

1. There exists a $(n+\ell)\times n$ matrix T with Rank $T = n$ such that $TF = AT + BH$.

2. All the eigenvalues of A are non-resonant with $\sigma(F)$.

3. 0 does not lie in the convex hull of $\sigma(F)$.

Then there exists a unique analytic solution $z = \psi(x)$ to the PDE (12) locally around $x = 0$ with $\frac{\partial \psi}{\partial x}(0) = T$.

<u>Remark 3:</u> From a practical point of view, the proposed observer design method requires the development of an approximate solution method for PDEs (7) and (12). In a similar spirit as in [8,11], one can develop a comprehensive power series solution scheme by taking advantage of the real analyticity property of all the functions involved. The calculations for the power series solution scheme can be executed, up to a finite truncation order, using symbolic computations software.

V. BIOREACTOR APPLICATION

Consider a typical bioreactor, where biochemical reactions take place, resulting in biomass production and substrate consumption following Monod kinetics:

$$\begin{aligned} \frac{dx}{dt} &= -Dx + \frac{\mu_{\max} s}{K+s} x \\ \frac{ds}{dt} &= D(s_f - s) - \frac{1}{Y}\frac{\mu_{\max} s}{K+s} x \end{aligned} \tag{18}$$

where x is the biomass concentration, s the substrate concentration, s_f the inlet substrate concentration, D the dilution rate, K a reaction constant, Y the yield coefficient and $\mu_{\max}$ the maximal specific growth rate. The biomass x is measurable on line, but the measurement could be subjected to a systematic error w. This is assumed to remain constant over a certain period of time, but potentially undergoing step changes. The objective is to estimate both the bioreactor's state and the systematic error (step disturbance) w. Therefore the dynamic system under consideration is:

$$\begin{aligned} \frac{dx}{dt} &= -Dx + \frac{\mu_{max} s}{K+s} x \\ \frac{ds}{dt} &= D(s_f - s) - \frac{1}{Y}\frac{\mu_{max} s}{K+s} x \\ \frac{dw}{dt} &= 0 \\ y &= x + w \end{aligned} \tag{19}$$

and the objective is to design a full-order observer for this system, to estimate the unmeasured substrate concentration s and the error w, following the method developed in the previous section.

For the design of the nonlinear observer, the following PDE needs to be solved:

$$\begin{aligned} &\frac{\partial \psi}{\partial x}(x,s) \left[-Dx + \frac{\mu_{max} s}{K+s} x\right] \\ &+ \frac{\partial \psi}{\partial s}(x,s) \left[D(s_f - s) - \frac{1}{Y}\frac{\mu_{max} s}{K+s} x\right] \\ &= A\psi(x,s) + Bx \end{aligned} \tag{20}$$

around a reference equilibrium point (x_s, s_s, w_s). Once (20) is solved, the nonlinear observer is given by:

$$\begin{aligned} \frac{d\hat{x}}{dt} &= -D\hat{x} + \frac{\mu_{max}\ \hat{s}}{K+\hat{s}}\hat{x} + L_x(\hat{x},\hat{s},\hat{w})(y - \hat{x} - \hat{w}) \\ \frac{d\hat{s}}{dt} &= D(s_f - \hat{s}) - \frac{1}{Y}\frac{\mu_{max}\ \hat{s}}{K+\hat{s}}\hat{x} + L_s(\hat{x},\hat{s},\hat{w})(y - \hat{x} - \hat{w}) \\ \frac{d\hat{w}}{dt} &= L_w(\hat{x},\hat{s},\hat{w})(y - \hat{x} - \hat{w}) \end{aligned} \tag{21}$$

where

$$\begin{bmatrix} L_x(\hat{x},\hat{s},\hat{w}) \\ L_s(\hat{x},\hat{s},\hat{w}) \\ L_w(\hat{x},\hat{s},\hat{w}) \end{bmatrix} = \begin{bmatrix} \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial s} & \frac{\partial \theta}{\partial w} \end{bmatrix}^{-1} B \tag{22}$$

and where $\theta(x,s,w) = \psi(x,s) + \Omega w$ with $\Omega = -A^{-1}B$.

A MAPLE code has been written, which solves the PDE (20) up to a finite truncation order N, calculates the observer gains via (22) and conducts numerical simulations, in order to test the observer (21).

In the present study, the following parameter values were used in all simulations:

s_f=50.0 , D=0.4 , K=2.0 , Y=0.5 , μ_{max}=0.9

and the reference equilibrium is:

x_s=24.2 , s_s=1.6 , w_s=0.0

which corresponds to zero sensing error.

The Jacobian of the dynamics of (19), evaluated at the reference equilibrium forms an observable pair with $\begin{bmatrix}1 & 0 & 1\end{bmatrix}$, i.e. system (19) is linearly observable.

The process (18) and the observer (21) were simulated with the following initial conditions:

$$x(0)=20.0, \qquad \hat{x}(0)=22.0$$
$$s(0)=7.0, \qquad \hat{s}(0)=3.0$$
$$w(0)=1.0, \qquad \hat{w}(0)=0.0$$

This accounts for a unit step change in the sensing error, in the presence of non-equilibrium initial conditions.

Figures 1-3 depict the effect of speed of the error dynamics for the same truncation order. Three different sets of eigenvalues were used, 'slow' (-0.05,-1.5,-3.0), 'medium-speed' (-0.35,-0.45,-3.9) and 'fast' (-1.5,-3.0,-3.0), which are all non-resonant with the linearization of (19). In all cases shown, the truncation order was N=5.

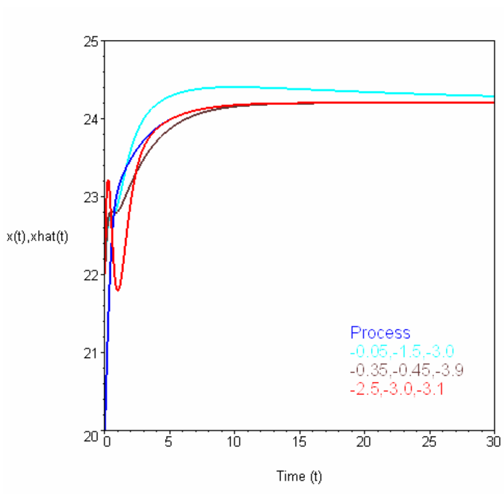

Fig. 1: True and estimated biomass concentration for different sets of eigenvalues

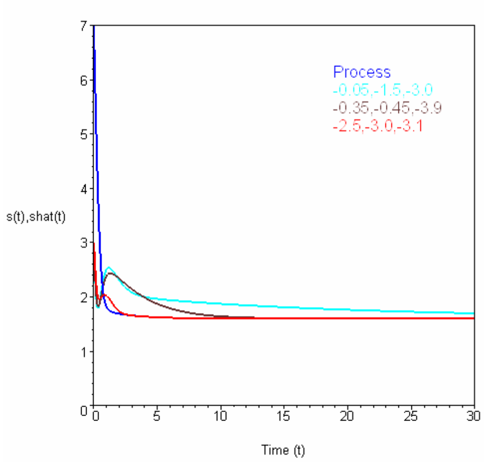

Fig. 2: True and estimated substrate concentration for different sets of eigenvalues

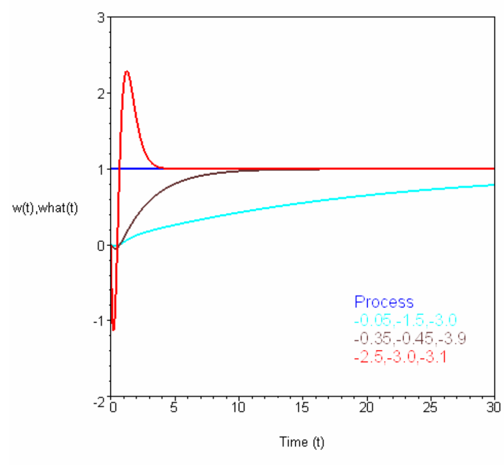

Fig. 3: True and estimated sensing error for different sets of eigenvalues

From Figures 1-3, one can see that the response of the observer with 'fast' eigenvalues converges to the process response very fast, but with significant deviations during the transient period, while the case of 'medium speed' eigenvalues shows a rapid response as well, but with smaller deviations. On the other hand, the observer with the 'slow' eigenvalues gives rise to a very slow approach of the error to zero.

The selection of error dynamics eigenvalues is, of course, application-dependent. If large short-lived errors can be tolerated and the settling time for the error is the most important performance parameter, fast eigenvalues may be preferable. Notice however, that using eigenvalues of higher speed than the 'fast' eigenvalues shown here, results in responses with excessively large deviations in the transient period, which appear unacceptable for all practical purposes.

Also, it must be noted that it is not only the "speed" of the eigenvalues (as measured by the eigenvalue that is closest to the imaginary axis) that affects the performance of the observer, but also the relative magnitudes of the eigenvalues. In the particular problem, it was found that using eigenvalues that are reasonably spread out, generally yields better performance than eigenvalues of similar magnitude.

Figures 4-6 depict the effect of truncation order on the observer response for the 'medium-speed' eigenvalues. It is seen that numerical convergence is achieved for N=3 and above. The effect of truncation order has also been studied for 'slow' and 'fast' eigenvalues. The results are not shown here for brevity. The results for 'slow' eigenvalues were quite similar in terms of numerical convergence, whereas the 'fast' ones converged at higher truncation order N=4.

The case N=1 needs special attention, since it corresponds to a constant gain observer, with gains being equal to what a linear design would have given for the linearized system. We see from Figures 4-6 that the constant-gain observer is significantly inferior to the nonlinear observer. The same behavior was found in the case of 'slow' eigenvalues. On the other hand, as the speed of eigenvalues increases, the response of the constant gain observer gets closer to the one of the nonlinear observer, while at the same time performance deteriorates due to larger deviations in the transient period.

Further simulation results (not shown) have studied the effect of size of sensing error. It was found that, as the size of the sensing error decreases, convergence with respect to N becomes faster and the discrepancy between nonlinear observer and constant-gain observer becomes smaller, because the problem becomes less nonlinear.

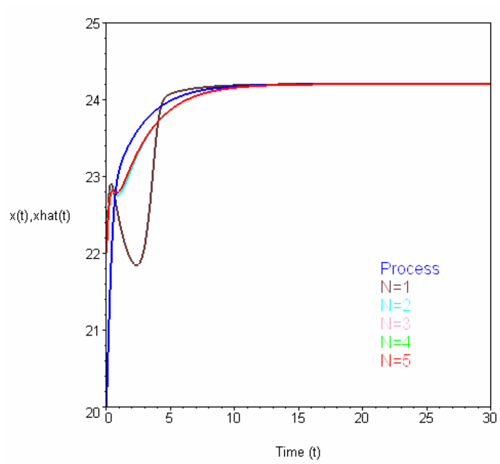

Fig. 4: True and estimated biomass concentration for different truncation orders for the 'medium-speed' eigenvalues

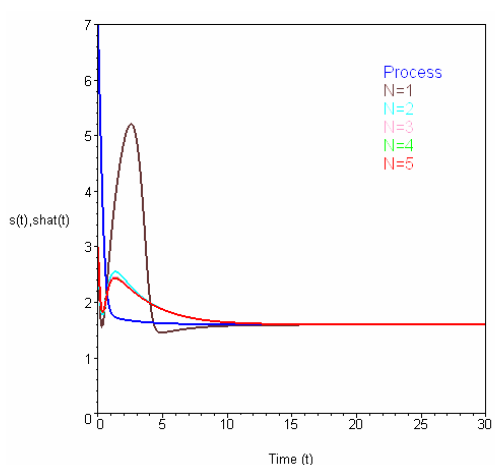

Fig. 5: True and estimated substrate concentration for different truncation orders for the 'medium-speed' eigenvalues

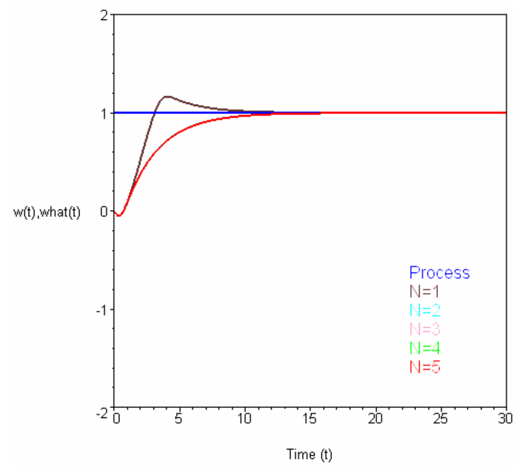

Fig. 6: True and estimated sensing error for different truncation orders for the 'medium-speed' eigenvalues

VI. CONCLUDING REMARKS

A new design framework for nonlinear observers capable of offering reliable concurrent estimates of the process state variables, along with key unknown process or sensor disturbances, was presented. In particular, the proposed nonlinear observer has a state-dependent gain that can be computed from the solution of a system of singular first-order PDEs. Within the proposed design framework, both state and disturbance estimation errors converge to zero with assignable rates.

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