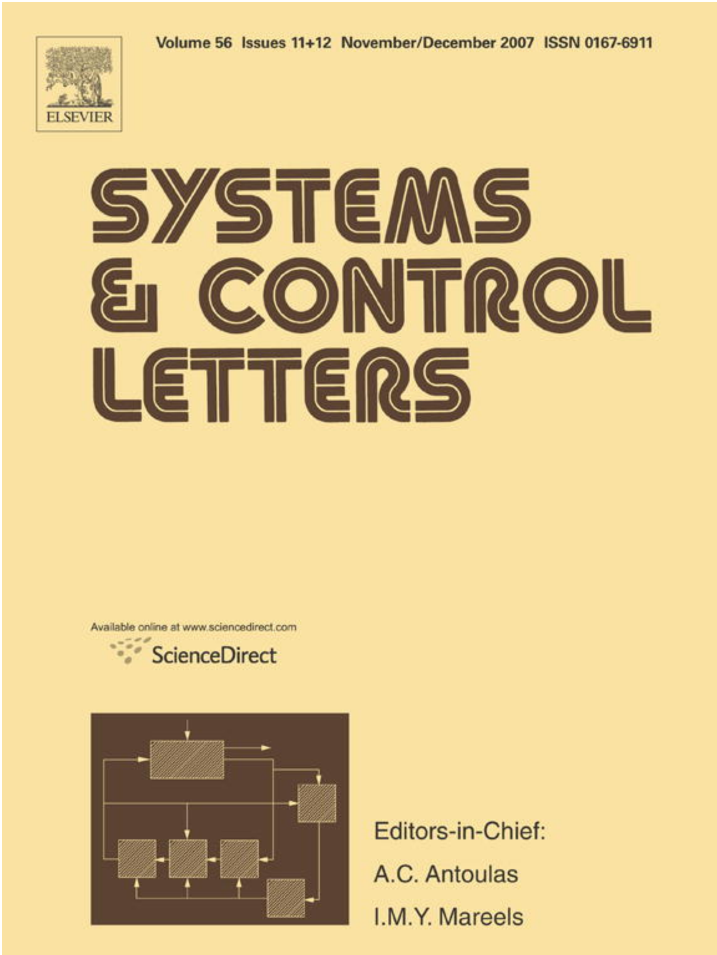
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Nonlinear observer design for state and disturbance estimation

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Abstract

A new systematic framework for nonlinear observer design that allows the concurrent estimation of the process state variables together with key unknown process or sensor disturbances is proposed. The nonlinear observer design problem is addressed within a similar methodological framework as the one introduced in [N. Kazantzis, C. Kravaris, Nonlinear observer design using Lyapunov's auxiliary theorem, Systems Control Lett. 34 (1998) 241; A.J. Krener, M. Xiao, Nonlinear observer design in the Siegel domain, SIAM J. Control Optim. 41 (2002) 932.] for state estimation purposes only. From a mathematical standpoint, the problem under consideration is addressed through a system of first-order singular PDEs for which a rather general set of solvability conditions is derived. A nonlinear observer is then designed with a state-dependent gain that is computed from the solution of the system of singular PDEs. Under the aforementioned conditions, both state and disturbance estimation errors converge to zero with assignable rates. The convergence properties of the proposed nonlinear observer are tested through simulation studies in an illustrative example involving a biological reactor.

Keywords: Nonlinear observers; State estimation; Disturbance estimation; Bioreactors

1. Introduction

Technical limitations and/or prohibitively high cost associated with current sensor technology entail the non-availability of all process state variables for direct on-line measurements. Furthermore, key process parameters frequently represent unknown or poorly known time-varying disturbances [3]. The operation of sensing devices is also subject to external disturbances, involving, for example, a sudden or gradual decalibration of the instrument. Therefore, there is an essential need for an accurate estimation of the unmeasurable process state variables together with key process or sensor disturbances [6,8,19]. In the case of linear systems, both the well-known Kalman filter and its deterministic analogue realized by Luenberger's observer [3] offer a comprehensive solution to the problem. However, the nonlinear observer design problem is much more challenging and has received a considerable amount of attention in the literature [1,2,4,6–20]. The present research work aims at the development of a new framework for nonlinear observer design that allows the concurrent estimation of the state variables, together with key unknown process or sensor disturbances. In particular, the nonlinear observer design problem is formulated and addressed within a similar methodological framework as the one first introduced in [12,16] for state estimation purposes only, and from a mathematical standpoint, via a system of first-order singular PDEs for which a rather general set of necessary and sufficient conditions for solvability is derived. A nonlinear observer is then designed that possesses a state-dependent gain computed from the solution of the system of PDEs. Under the above conditions, it is proven that both state and disturbance estimation errors converge to zero with assignable rates. The proposed method is evaluated in an illustrative bioreactor example through simulation studies.

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2. Problem formulation

Consider a dynamic system

$$\dot{x} = f(x, w),$$
$$y = h(x, w) \tag{1}$$

that represents the dynamics of a process, where x is the process state vector, y is the vector of measurements and w is the vector of unmeasurable process or sensor disturbances. The dynamics of the disturbances is governed by the exosystem

$$\dot{w} = s(w). \tag{2}$$

Notice that disturbance models such as (2) are traditionally considered in output regulation problems [5]. The problem of *state and disturbance estimation* becomes a pure state estimation problem when one considers the extended system (1)–(2), where $\begin{bmatrix} x \\ w \end{bmatrix}$ is the extended system's state vector, which must be estimated via an appropriately designed observer.

The approach that will be taken in this work is the *observer error linearization* approach [12,16]. Generally speaking, the degree of difficulty of the observer design problem depends on the nature of the eigenvalues of the linearization of the extended system, in particular whether their convex hull includes the origin (spectrum is in the Siegel domain) or does not include the origin (spectrum is in the Poincaré domain).

The following definitions will be needed for the rest of the paper:

- Given a set of eigenvalues $\lambda_1, \ldots, \lambda_n$, a complex number μ is said to be *nonresonant* with this set of eigenvalues if it is not related with them through any relation of the form $\mu = \sum_{i=1}^{n} m_i \lambda_i$, where $m_1, \ldots, m_n$ are nonnegative integers not all zero.
- Given a set of eigenvalues $\lambda_1, \ldots, \lambda_n$, a complex number μ is said to be *of type* (C, v) with respect to this set of eigenvalues if there exist constants $C > 0$ and $v > 0$ such that $|\mu - \sum_{i=1}^{n} m_i \lambda_i| \geqslant \frac{C}{(\sum_{i=1}^{n} m_i)^v}$ for any nonnegative integers $m_1, \ldots, m_n$ that are not all zero.

3. Nonlinear observer design

Consider the augmented system

$$\dot{x} = f(x, w),$$
$$\dot{w} = s(w),$$
$$y = h(x, w), \tag{3}$$

where $f : R^n \times R^\ell \to R^n, s : R^\ell \to R^\ell, h : R^n \times R^\ell \to R^\rho$ are real analytic functions, with $f(0, 0) = 0$, $s(0) = 0$, $h(0, 0) = 0$. Similarly as in [12,16], a local diffeomorphism $z = \theta(x, w)$ is sought that maps system (3) into

$$\dot{z} = Az + \beta(y), \tag{4}$$

where A is a $(n+\ell)\times(n+\ell)$ matrix and $\beta : R^\rho \to R^{n+\ell}$ is a real analytic function with $\beta(0)=0$. As long as such a transformation can be found, (4) can be used as observer dynamics and the inverse transformation can be used to reconstruct the system states:

$$\dot{\hat{z}} = A\hat{z} + \beta(y),$$
$$\begin{bmatrix} \hat{x} \\ \hat{w} \end{bmatrix} = \theta^{-1}(\hat{z}). \tag{5}$$

It turns out that the unknown transformation map θ must satisfy the following system of singular PDEs [12,16]:

$$\frac{\partial\theta}{\partial x}(x, w) f(x, w) + \frac{\partial\theta}{\partial w}(x, w)s(w) = A\theta(x, w) + \beta(h(x, w)). \tag{6}$$

Therefore, the problem of interest reduces to the study of PDEs (6) and the properties of the solution. The following propositions are direct consequences of the results in [12] and [16,17], respectively.

Proposition 1. *Let $f : R^n \times R^\ell \to R^n$, $s : R^\ell \to R^\ell$, $h : R^n \times R^\ell \to R^\rho$ and $\beta : R^\rho \to R^{n+\ell}$ be real analytic vector functions with $f(0, 0) = 0$, $s(0) = 0$, $h(0, 0) = 0$, $\beta(0) = 0$ and $F = (\partial f/\partial x)(0, 0)$, $P = (\partial f/\partial w)(0, 0)$, $S = (\partial s/\partial w)(0)$, $H=(\partial h/\partial x)(0, 0)$, $Q=(\partial h/\partial w)(0, 0)$, $B=(\partial\beta/\partial y)(0)$. Denote by $\sigma(F)$ and $\sigma(S)$ the spectra of S and F, respectively.*

Suppose:

1. *There exists an invertible $(n + \ell) \times (n + \ell)$ matrix T such that $T\begin{bmatrix} F & P \\ 0 & S \end{bmatrix} = AT + B[H \;\; Q]$.*
2. *All the eigenvalues of A are nonresonant with $\sigma(F) \cup \sigma(S)$.*
3. *0 does not lie in the convex hull of $\sigma(F) \cup \sigma(S)$.*

Then there exists a unique analytic solution $z = \theta(x, w)$ to PDE (6) locally around $(x, w) = (0, 0)$. The solution has the property that $[(\partial\theta/\partial x)(0, 0) \;\; (\partial\theta/\partial w)(0, 0)] = T$ and so θ is a local diffeomorphism.

Proposition 2. *Under the notations of Proposition 1, suppose*:

1. *There exists an invertible $(n + \ell) \times (n + \ell)$ matrix T such that $T\begin{bmatrix} F & P \\ 0 & S \end{bmatrix} = AT + B[H \;\; Q]$.*
2. *There exist $C > 0$, $v > 0$ such that all the eigenvalues of A are of type (C, v) with respect to $\sigma(F) \cup \sigma(S)$.*
3. *There exist $C > 0$, $v > 0$ such that all the eigenvalues of F and S are of type (C, v) with respect to $\sigma(F) \cup \sigma(S)$.*

Then there exists a unique analytic solution $z = \theta(x, w)$ to PDE (6) locally around $(x, w) = (0, 0)$. The solution has the property that $[(\partial\theta/\partial x)(0, 0) \;\; (\partial\theta/\partial w)(0, 0)] = T$ and so θ is a local diffeomorphism.

Remark 1. Assumptions 1 and 2 of either proposition imply that $\left([H \;\; Q], \begin{bmatrix} F & P \\ 0 & S \end{bmatrix}\right)$ is an observable pair. On the other hand, if $\left([H \;\; Q], \begin{bmatrix} F & P \\ 0 & S \end{bmatrix}\right)$ is an observable pair, it is always possible to find matrices A, B, T which satisfy the matrix equation of assumption 1, with T invertible and A having prescribed eigenvalues.

Remark 2. Under the assumption that $\sigma(F)$ and $\sigma(S)$ are disjoint sets, it is possible to show that the pair of composite matrices $\left([H \;\; Q], \begin{bmatrix} F & P \\ 0 & S \end{bmatrix}\right)$ is observable if and only if the following conditions hold:

(a) (H, F) is an observable pair.
(b) $(HR + Q, S)$ is an observable pair, where R is the solution of $RS - FR = P$.

Condition (b) implies that no eigenvalue of S is a transmission zero of (F, P, H, Q).

It should be noted that observer (5) can be expressed in the original coordinates via the inverse transformation θ^{-1}, so that $\hat{x}$ and $\hat{w}$ explicitly represent the observer states:

$$\dot{\hat{x}} = f(\hat{x}, \hat{w}) + L_x(\hat{x}, \hat{w})\{\beta(y) - \beta(h(\hat{x}, \hat{w}))\},$$
$$\dot{\hat{w}} = s(\hat{w}) + L_w(\hat{x}, \hat{w})\{\beta(y) - \beta(h(\hat{x}, \hat{w}))\}, \tag{7}$$

where $\begin{bmatrix} L_x(\hat{x},\hat{w}) \\ L_w(\hat{x},\hat{w}) \end{bmatrix} = [(\partial\theta/\partial x)(\hat{x}, \hat{w}) \;\; (\partial\theta/\partial w)(\hat{x}, \hat{w})]^{-1}$.

Finally, it should be noted that the estimation error in the transformed coordinates follows linear dynamics, governed by the arbitrarily selected matrix A (design parameter):

$$\frac{\mathrm{d}}{\mathrm{d}t}[\theta(x, w) - \theta(\hat{x}, \hat{w})] = A[\theta(x, w) - \theta(\hat{x}, \hat{w})].$$

4. Nonlinear observer design for state and sensing error estimation

A special case of the problem under consideration is the *state and sensing error estimation problem*, where the disturbances affect the sensing devices only, and in an additive way:

$$\dot{x} = f(x),$$
$$\dot{w} = s(w),$$
$$y = h(x) + q(w). \tag{8}$$

Also, suppose that for the design of the observer, linear output injection $\beta(y) = By$ is used, where B is an $(n + \ell) \times \rho$ matrix. Then, the system of PDEs (6) becomes

$$\frac{\partial\theta}{\partial x}(x, w) f(x) + \frac{\partial\theta}{\partial w}(x, w) s(w) = A\theta(x, w) + B(h(x) + q(w)). \tag{9}$$

The solution of (9) can be expressed as

$$\theta(x, w) = \psi(x) + \omega(w), \tag{10}$$

where ψ and ω satisfy the following system of PDEs:

$$\frac{\partial\psi}{\partial x}(x) f(x) = A\psi(x) + Bh(x), \tag{11}$$

$$\frac{\partial\omega}{\partial w}(w) s(w) = A\omega(w) + Bq(w). \tag{12}$$

In this way, the system of PDEs for θ (9) is partitioned into two decoupled sub-systems of PDEs of smaller dimension, and, therefore, the computational effort is significantly reduced. PDE (11) is exactly the PDE for the observer design for the disturbance-free part of the system, whereas (12) is the corresponding observer PDE for the disturbance dynamics. For $\theta(x, w)$ of the form (10), with ψ and ω being solutions of (11) and (12), observer (7) takes the form

$$\dot{\hat{x}} = f(\hat{x}) + L_x(\hat{x}, \hat{w})(y - h(\hat{x}) - q(\hat{w})),$$
$$\dot{\hat{w}} = s(\hat{w}) + L_w(\hat{x}, \hat{w})(y - h(\hat{x}) - q(\hat{w})), \tag{13}$$

where the corresponding gains are given by the following expressions:

$$\begin{bmatrix} L_x(\hat{x}, \hat{w}) \\ L_w(\hat{x}, \hat{w}) \end{bmatrix} = \begin{bmatrix} \frac{\partial\psi}{\partial x}(\hat{x}) & \frac{\partial\omega}{\partial w}(\hat{w}) \end{bmatrix}^{-1} B. \tag{14}$$

It should be noted that in most engineering applications, there are two further simplifications:

(i) The disturbances w are considered to be prototype disturbances (e.g. steps, ramps, sine waves, etc.) that follow linear dynamics, which means $s(\cdot)$ and $q(\cdot)$ are linear functions:

$$s(w) = Sw,$$
$$q(w) = Qw, \tag{15}$$

where S and Q are matrices of appropriate dimensions. Then the solution to PDE (12) is also a linear function:

$$\omega(w) = \Omega w, \tag{16}$$

where Ω is the solution of the matrix equation $\Omega S - A\Omega = BQ$.

(ii) The dynamics of the process states $\dot{x} = f(x)$ is locally exponentially stable, which means that the eigenvalues of its linearization are in the Poincaré domain. Then the results in [12] lead to the following:

Proposition 3. *Let $f : R^n \to R^n$ and $h : R^n \to R^\rho$ be real analytic vector functions with $f(0) = 0$, $h(0) = 0$ and $F = (\partial f/\partial x)(0)$, $H = (\partial h/\partial x)(0)$.*

Suppose:

1. *There exists an $(n + \ell) \times n$ matrix T with Rank $T = n$ such that $TF = AT + BH$.*
2. *All the eigenvalues of A are nonresonant with $\sigma(F)$.*
3. 0 *does not lie in the convex hull of $\sigma(F)$.*

Then there exists a unique analytic solution $z = \psi(x)$ to PDE (11) *locally around $x = 0$ with $(\partial\psi/\partial x)(0) = T$.*

Remark 3. From a practical point of view, the proposed observer design method requires the development of an approximate solution method for PDEs (6) and (11). In a similar spirit as in [12,16], one can develop a comprehensive power-series solution scheme by taking advantage of the real analyticity property of all the functions involved. The calculations for the power-series solution scheme can be executed, up to a finite truncation order, using symbolic computations software. Furthermore, convergence properties may be further enhanced by adopting ideas presented in [16,18].

5. Bioreactor application

Consider a typical bioreactor with biomass production and substrate consumption following Monod kinetics [3]:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -Dx + \frac{\mu_{\max} s}{K+s}x,$$
$$\frac{\mathrm{d}s}{\mathrm{d}t} = D(s_f - s) - \frac{1}{Y}\frac{\mu_{\max} s}{K+s}x, \tag{17}$$

where x is the biomass concentration, s the substrate concentration, s_f the inlet substrate concentration, D the dilution rate, K a reaction constant, Y the yield coefficient and $\mu_{\max}$ the maximal specific growth rate. It is assumed that the process parameters satisfy $\mu_{\max}/D > 1 + K/s_f$, which guarantees the existence of a unique positive equilibrium for the bioreactor dynamics. Moreover, it guarantees that the equilibrium is exponentially stable, hence the spectrum of the linearization of (17) is in the Poincaré domain.

The biomass x is measurable on line, but the measurement could be subjected to a systematic error w. This is assumed to remain constant over a certain period of time, but potentially undergoing step changes. The objective is to estimate both the reactor's state and the systematic error w. Therefore, the extended system under consideration is

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -Dx + \frac{\mu_{\max} s}{K+s}x,$$
$$\frac{\mathrm{d}s}{\mathrm{d}t} = D(s_f - s) - \frac{1}{Y}\frac{\mu_{\max} s}{K+s}x,$$
$$\frac{\mathrm{d}w}{\mathrm{d}t} = 0,$$
$$y = x + w, \tag{18}$$

and the objective is to design an observer to estimate the unmeasured substrate concentration s and the error w, following the proposed method. For the design of the nonlinear observer, the following PDE needs to be solved:

$$\frac{\partial \psi}{\partial x}(x,s)\left[-Dx + \frac{\mu_{\max} s}{K+s}x\right] + \frac{\partial \psi}{\partial s}(x,s)\left[D(s_f - s) - \frac{1}{Y}\frac{\mu_{\max} s}{K+s}x\right] = A\psi(x,s) + Bx. \tag{19}$$

Once (19) is solved, the nonlinear observer is given by

$$\frac{\mathrm{d}\hat{x}}{\mathrm{d}t} = -D\hat{x} + \frac{\mu_{\max}\hat{s}}{K+\hat{s}}\hat{x} + L_x(\hat{x},\hat{s},\hat{w})(y - \hat{x} - \hat{w}),$$
$$\frac{\mathrm{d}\hat{s}}{\mathrm{d}t} = D(s_f - \hat{s}) - \frac{1}{Y}\frac{\mu_{\max}\hat{s}}{K+\hat{s}}\hat{x} + L_s(\hat{x},\hat{s},\hat{w})(y - \hat{x} - \hat{w}),$$
$$\frac{\mathrm{d}\hat{w}}{\mathrm{d}t} = L_w(\hat{x},\hat{s},\hat{w})(y - \hat{x} - \hat{w}), \tag{20}$$

where

$$\begin{bmatrix} L_x(\hat{x},\hat{s},\hat{w}) \\ L_s(\hat{x},\hat{s},\hat{w}) \\ L_w(\hat{x},\hat{s},\hat{w}) \end{bmatrix} = \begin{bmatrix} \frac{\partial\theta}{\partial x} & \frac{\partial\theta}{\partial s} & \frac{\partial\theta}{\partial w} \end{bmatrix}^{-1} B \tag{21}$$

and where $\theta(x,s,w) = \psi(x,s) + \Omega w$ with $\Omega = -A^{-1}B$.

A MAPLE code has been written, which solves PDE (19) around an equilibrium point of (18) up to truncation order N, calculates the observer gains via (21) and simulates observer (20).

In the present study, the following parameter values were used in all simulations:

$$s_f = 50.0, \quad D = 0.4, \quad K = 2.0, \quad Y = 0.5, \quad \mu_{\max} = 0.9$$

and the reference equilibrium point is

$$x_s = 24.2, \quad s_s = 1.6, \quad w_s = 0.0$$

which corresponds to zero sensing error.

The Jacobian of the dynamics of (18), evaluated at the reference equilibrium, forms an observable pair with [1 0 1], i.e. system (18) is linearly observable.

Process (18) and observer (20) were simulated with the following initial conditions:

$$x(0) = 20.0, \quad \hat{x}(0) = 22.0,$$
$$s(0) = 7.0, \quad \hat{s}(0) = 3.0,$$
$$w(0) = 1.0, \quad \hat{w}(0) = 0.0.$$

This accounts for a unit step change in the sensing error, in the presence of non-equilibrium initial conditions.

5.1. *Effect of choice of eigenvalues for the error dynamics*

Fig. 1 depicts the effect of speed of the error dynamics for the same truncation order. For brevity we present the results only for the sensing error. Three different sets of eigenvalues were used, 'slow' (−0.05, −1.5, −3.0), 'medium speed' (−0.35, −0.45, −3.9) and 'fast' (−2.5, −3.0, −3.1), which are all nonresonant with the linearization of (17). In all cases shown, the truncation order was $N = 5$.

From Fig. 1, one can see that the response of the observer with 'fast' eigenvalues converges to the process response very fast, but with significant deviations during the transient period, while the case of 'medium-speed' eigenvalues shows a reasonably rapid response, with smaller deviations. On the other hand, the observer with the 'slow' eigenvalues gives rise to a very slow approach of the error to zero. The same behavior is observed for both the biomass and substrate concentrations. Throughout the rest of the paper, the focus will be on the sensing error and additional comments regarding the other two states will be made where necessary.

The selection of error dynamics eigenvalues is, of course, application dependent. If large short-lived errors can be tolerated and the settling time for the error is the most important performance parameter, fast eigenvalues may be preferable. Notice, however, that using eigenvalues of higher speed than the 'fast' eigenvalues shown here, results in responses with excessively

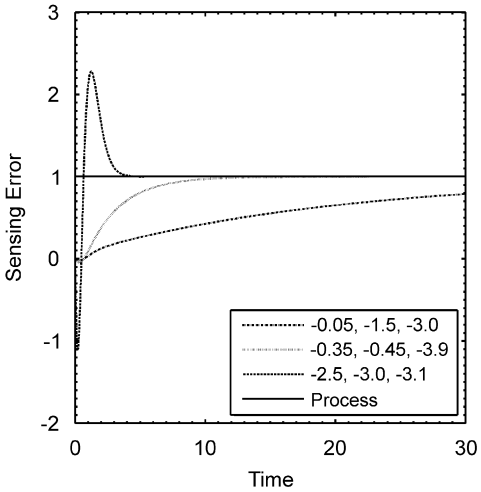

Fig. 1. True and estimated sensing error for different sets of eigenvalues for truncation order $N = 5$.

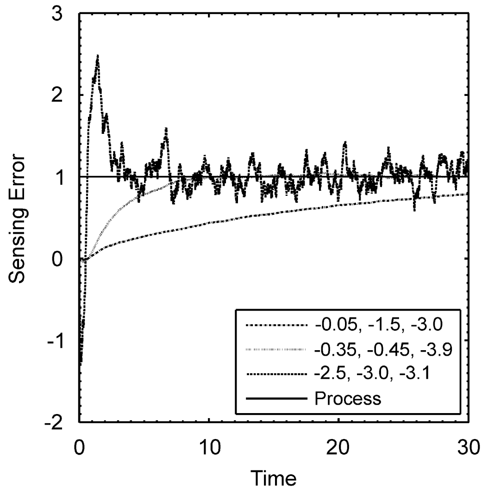

Fig. 2. True and estimated sensing error in the presence of white noise of standard deviation 0.4, for different sets of eigenvalues and truncation order $N = 5$.

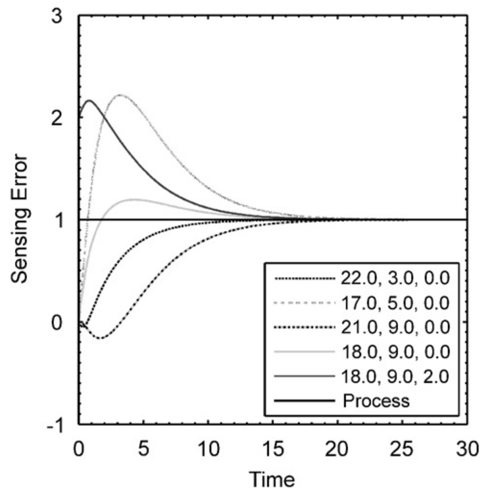

Fig. 3. True and estimated sensing error for different initial conditions for the 'medium-speed' eigenvalues and truncation order $N = 5$.

large deviations in the transient period. Also, it must be noted that it is not only the 'speed' of the eigenvalues that affects the performance of the observer, but also the relative magnitudes of the eigenvalues. In the particular problem, using eigenvalues that are reasonably spread out yields better performance than with eigenvalues of similar magnitude.

5.2. *Effect of truncation order*

In all simulation runs, calculations were performed for different truncation orders N of the power-series solution algorithm for PDE (19). This was necessary in order to test numerical convergence of the approximation scheme with respect to N. It was found that for the 'slow' and 'medium' eigenvalues, numerical convergence of the observer responses was achieved for $N = 3$. Numerical convergence for the 'fast' eigenvalues was achieved at a higher truncation order $N = 4$. Simulation results for different truncation orders N are omitted for brevity.

The case $N = 1$ is of special interest, since it corresponds to a constant-gain observer, with gains being equal to what a linear design would have given for the linearized system. For the previously considered sets of 'slow' and 'medium' eigenvalues and initial conditions, the performance of the constant-gain observer was found to be significantly inferior to the nonlinear observer. Increasing the speed of eigenvalues, the response of the constant-gain observer came closer to the one of the nonlinear observer, while at the same time performance deteriorated due to larger deviations in the transient period. In simulation runs with other initial conditions, there were cases where the response of the constant-gain observer blew up.

5.3. *Effect of measurement noise*

In addition to systematic error in the sensing device, noise is expected to be present in the measurement signal. In this case, the observer is driven by $y = x + w + v$, where w, as before, represents the systematic error in the measurement and v is the measurement noise.

In order to test the ability of the proposed observer to perform well in the presence of noise, the foregoing numerical calculations were all repeated by adding white noise to the measurement signal. White noise was simulated using normally distributed random numbers of zero mean and 0.4 standard deviation.

Fig. 2 depicts the responses of the proposed nonlinear observer, under the same sets of eigenvalues and initial conditions as before, but with the simulated white noise being added to the measurement signal. Comparing these responses with the ones in the absence of noise (Fig. 1), it is seen that this observer is capable of performing well under this level of noise.

The observer acts as a filter that attenuates measurement noise. Increasing speed of eigenvalues generally gives rise to larger observer gains and therefore higher sensitivity to noise (cf. Fig. 2). For the given level of noise, the 'medium-speed' eigenvalues give the most reasonable response, combining speed and noise rejection capability.

5.4. *Effect of initial conditions of the observer*

Finally, the effect of the initial conditions on the observer responses was studied. Five different sets of initial conditions were tried, including the ones previously used. Fig. 3 depicts the estimates of the sensing error for the 'medium-speed' eigenvalues with truncation order $N = 5$, for the different initial conditions. It is observed that the observer performs well for all different cases.

It is important to note that for three of the initial condition sets, $(17.0, 5.0, 0.0)$, $(18.0, 9.0, 0.0)$ and $(18.0, 9.0, 2.0)$, the linear design ($N = 1$) led to unstable observer response.

6. Conclusion

A new design framework for nonlinear observers capable of offering reliable concurrent estimates of the process state variables, along with key unknown process or sensor disturbances, was developed. In particular, the proposed nonlinear observer has a state-dependent gain that can be computed from the solution of a system of singular first-order PDEs. Within the proposed design framework, both state and disturbance estimation errors converge to zero with assignable rates.

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