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Application of An Efficient Nonlinear Filter*

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Abstract- *In this paper we present an application of a new filtering technique based on geometric linearization and asymptotic analysis. The technique is compared to the conventional extended Kalman filter to demonstrate its computational efficiency.*

Introduction

While the theory of linear filtering has been well developed and understood, practical nonlinear filtering has typically relied on heuristic techniques. One of these is the extended Kalman filter which is based on a linear approximation of the system equations around a trajectory. The asymptotic geometric nonlinear filtering technique was developed by Krener in [5]. It is based on the so-called observer normal form which linearizes the dynamics by a change of state coordinates and output injection. The development of the observer normal form is due to Krener and Isidori [1], Krener and Respondek [2], Bestle and Zeitz [8], Zeitz [11,12], Fritz and Keller [9], Keller [6,7], and Li and Tao [10].

The approach is geometric and consists of finding a change of coordinates, in general nonlinear, such that the equations of the system transform to their most linear form. Once the system is in "nearly linear" form it will be possible to apply asymptotically the theory of linear filtering. This has many advantages. In fact it is possible to formally define optimality and asymptotic stability. Moreover, the gains of the filter may be computed off-line because the Riccati differential equation is independent of the states. This reduces the on-line computational burden of the filter.

The only drawback of the technique is that the change of coordinates generally requires a very heavy algebraic computational effort. One solution to this is to use already existing software packages for symbolic computations. Naturally the technique is not applicable to all nonlinear systems; otherwise we would have discovered that everything in nature is linear in some appropriate coordinate set. The class of "nearly linearizable" systems is substantial, and gives to the method a certain flavor of generality.

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In this paper we will illustrate the technique for a specific example. We will estimate the height, the velocity and the ballistic coefficient of a falling object in an atmosphere with variable density. We will also implement the extended Kalman filter for the same problem and compare the two filtering techniques in terms of performance and computation time. Throughout the paper we will refer to the geometric asymptotic nonlinear filter, the new technique, as the GANF and to the extended Kalman filter as the EKF.

1- Brief description of the EKF and the GANF techniques

We say that a given nonlinear system without input:

$$\dot{\zeta} = f(\zeta),$$
$$\dot{y} = h(\zeta). \tag{1}$$

where $\zeta \varepsilon \mathbf{R}^n$ and $\dot{y} \varepsilon \mathbf{R}^1$, is observable if it can be transformed into observable normal form. This corresponds, in some sense, to the property of observability for a linear system. Normal forms have the advantage of making transparent the effect of an input on the dynamics of the system. There are four such normal forms: Observer, observable, controller and controllable. In the present application, and in the case of nonlinear observers in general, we will have occasion to use only the first two:

Observable form:

$$\dot{x} = Ax - B\alpha(x)$$
$$\dot{y} = Cx \tag{2}$$

Observer form:

$$\dot{x} = Ax - \alpha(Cx)$$
$$\dot{y} = \gamma(Cx) \tag{3}$$

In some particular cases (in the application treated in this paper, for example) we will require a modified observer form.

Modified observer form:

$$\dot{x} = Ax - \alpha(Cx, CAx, \ldots, CA^{n-1}x)$$
$$\dot{y} = \gamma(Cx) \tag{4}$$

where A, B, C are the standard matrices of the Brunovsky canonical form. For the case of a three dimensional system with a single output:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{5}$$

The system (1) can be transformed into observable form if y and its first n-1 time derivatives are local coordinates on the state space.

A physical system will never satisfy exactly a set of differential equations like (1). In reality each of the states is affected by random process noise, the parameters are not known exactly, and the measurement of the output will be affected by observation noise due to the physical and technical limitations on the measurement procedure. The system (1) can then be expressed in its stochastic equivalent:

$$\begin{aligned} d\zeta &= f(\zeta)dt + Gdw \\ dy &= h(\zeta)dt + Ddv \\ \zeta(0) &= N(\zeta_0, P_0) \end{aligned} \tag{6}$$

Where w and v are standard Wiener processes. The covariances of the driving and measurement noises are, respectively, $Q = GG^T$ and $R = D^2$. For our work we will assume the measurement noise to be small, i.e. $D = \varepsilon^2$ where ε is a small parameter. Then the problem is to estimate the states of (6) at time t given the measurement of y at time less than or equal to t. It is clear that in the absence of noise and if (1) is observable the problem is easily solved because the states could be computed exactly as nonlinear functions of derivatives up to order n - 1 of the output.

We compute the estimates by introducing the filter:

$$\begin{aligned} d\hat{\zeta} &= f(\hat{\zeta})dt + g(\hat{\zeta})(dy - h(\hat{\zeta})dt) \\ d\hat{y} &= h(\hat{\zeta})dt \end{aligned} \tag{7}$$

The conventional extended Kalman filter technique computes the estimates from:

$$\begin{aligned} d\hat{\zeta} &= f(\hat{\zeta})dt + P(t)H^T(\hat{\zeta}, t)R^{-1}(dy - h(\hat{\zeta})dt) \\ d\hat{y} &= h(\hat{\zeta})dt \end{aligned} \tag{8}$$

where P is the solution to the Riccati differential equation

$$\begin{aligned} \dot{P} &= F(\hat{\zeta}, t)P + PF^T(\hat{\zeta}, t) + Q(t) - PH^T(\hat{\zeta}, t)R^{-1}H(\hat{\zeta}, t)P \\ P(t_0) &= P_0 \end{aligned} \tag{9}$$

and

$$H(\hat{\zeta}, t) = \left\{ \frac{\partial h(\zeta, t)}{\partial \zeta_j} \Big|_{\zeta = \hat{\zeta}} \right\} \tag{10}$$

$$F(\hat{\zeta}, t) = \left\{ \frac{\partial f_i(\zeta, t)}{\partial \zeta_j} \Big|_{\zeta = \hat{\zeta}} \right\} \tag{11}$$

and P, Q and R are, respectively, the covariance matrices of the estimate error, the process noise and the observation noise.

The GANF computes the filter in observer form coordinates. In these coordinates the system (6) can be written as:

$$\begin{aligned} dx &= (Ax - \alpha(y))dt + \bar{G}dw \\ dy &= Cxdt + \varepsilon^2 dv \\ x(0) &= N(x_0, \bar{P}_0) \end{aligned} \tag{12}$$

where, if J is the Jacobian of the change of coordinates, $\bar{G} = JG$. If the measurement noise is small, and $\alpha(y)$ is smooth enough, then $\alpha(y)$ is approximately equal to $\alpha(C\hat{x})$ and we can use the filter equations

$$d\hat{x} = A\hat{x}dt - \alpha(C\hat{x})dt + \bar{P}C^T R^{-1}(dy - Cx\,dt) \tag{13}$$

The dynamics of the error are given by:

$$d\tilde{x} = (A - \bar{P}C^T R^{-1} C)\,\tilde{x}\,dt + (\alpha(C\hat{x}) - \alpha(y))dt + \bar{G}dw - \bar{P}C^T R^{-1} C\varepsilon^2 dv \tag{14}$$

which can also be written as:

$$d\tilde{x} = (A^* - \bar{P}C^T R^{-1} C)\,\tilde{x}\,dt + (\alpha^*(C\hat{x}) - \alpha^*(y))dt + \bar{G}dw - \bar{P}C^T R^{-1} C\varepsilon^2 dv \tag{14'}$$

where

$$A^* = A + \frac{d\alpha(C\hat{x})}{dy}C$$

$$\alpha^*(x) = \alpha(x) - \frac{d\alpha(C\hat{x})}{dy}Cx$$

$\bar{P}$ is obtained from the solution to the Riccati differential equation

$$\begin{aligned} \dot{\bar{P}} &= A^*\bar{P} + \bar{P}A^{*T} + \bar{Q} - \bar{P}C^T R^{-1} C\bar{P} \\ \bar{P}(0) &= \bar{P}_0 \end{aligned} \tag{15}$$

where $\bar{Q} = \bar{G}\bar{G}^T$. The Riccati equation (15) is state independent and hence can be integrated off-line. The dynamics of the error are nearly linear up to output injection, and the covariance of $\tilde{x}$ asymptotically equals $\bar{P}(t)$. We have assumed, without loss of generality, that the output injection term can be expanded in a Taylor series starting from the second order term:

$$\overset{\bullet}{\alpha}(C\hat{x}) - \overset{\bullet}{\alpha}(y) = \frac{d^2\alpha(C\hat{x})}{dy^2}(C\tilde{x} + \epsilon^2 dv)^2 + O(3) \qquad (16)$$

In fact, we have combined the linear term of α in A^*. Then, if the second derivative of α is small and the dynamics of the filter is stable, then the output injection term can be neglected. Practically, we are requiring α to have a small "curvature" in some sense.

One should note that these two filters are not being compared on exactly the same grounds because the noise covariances of the system are assumed to be state independent in ζ coordinates for the EKF, and in x coordinates for the GANF. However, since $\bar{Q} = JQJ^T$ and R is invariant under the change of coordinates, the comparison is justified if the Jacobian of the change of coordinates J is nearly constant along the trajectory. The main advantage of the GANF over the EKF is that the Riccati equation (14) can be solved off-line. This allows the computation of $g(\hat{\zeta})$ off-line, while when using the EKF it is necessary to integrate (9) and compute (10) and (11) on-line. In particular if the problem can be transformed into observer form with $\alpha(y) = 0$ then the GANF is the optimal filter while the extended Kalman filter is generally not.

2- An Application of GANF

We consider a falling object in an atmosphere of varying density. The problem is to estimate the position, the velocity and the ballistic coefficient of the object. This problem has been discussed previously by Gelb [3] and Wishner et al [4].

The dynamic model consists of:

$$\begin{aligned}
\dot{y} &= \zeta_1 \\
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= -g + \rho(\zeta_1)\zeta_2^2\zeta_3 \\
\dot{\zeta}_3 &= 0. \\
\zeta_1(0) &= \zeta_1^0 \\
\zeta_2(0) &= \zeta_2^0 \\
\zeta_3(0) &= \zeta_3^0
\end{aligned} \qquad (17)$$

where ζ_1 is the vertical position, ζ_2 the velocity, ζ_3 the inverse of the ballistic coefficient and

$$\rho(\zeta_1) = \rho_0 \exp\left(-\frac{\zeta_1}{k_\rho}\right) \tag{18}$$

represents the variation of the atmospheric density with the height ζ_1. It can be verified that this system is observable according to the observability condition defined in section 1. In fact

$$\begin{aligned}
\operatorname{grad} \dot{y} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \\
\operatorname{grad} \ddot{y} &= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \\
\operatorname{grad} \dddot{y} &= \begin{bmatrix} -\rho(\zeta_1)\frac{\zeta_2^2}{k_\rho}\zeta_3 & \rho(\zeta_1)2\zeta_2\zeta_3 & \rho(\zeta_1)\zeta_2^2 \end{bmatrix}
\end{aligned} \tag{19}$$

span $\mathbf{R}^3$, except at the singular point $\zeta_2 = 0$ where, the object being stationary, it is impossible to observe any of its dynamics. Hence it is possible to write (17) in Observable form:

$$\begin{aligned}
\dot{y} &= \xi_1 \\
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= \xi_3 \\
\dot{\xi}_3 &= f_3(\xi) = -g\frac{\xi_2}{k_\rho} + \left(2\frac{g}{\xi_2} - \frac{\xi_2}{k_\rho}\right)\xi_3 + \frac{2}{\xi_2}\xi_3^2
\end{aligned} \tag{20}$$

The Jacobian of the transformation between ζ and ξ is

$$\frac{\partial \xi}{\partial \zeta}(\zeta) = \begin{bmatrix} 1. & 0 & 0 \\ 0 & 1 & 0 \\ -\rho\frac{\zeta_2^2\zeta_3}{k_\rho} & 2\rho\zeta_2\zeta_3 & \rho\zeta_2^2 \end{bmatrix} \tag{21}$$

Unfortunately (20) does not satisfy the conditions for the transformation to observer form, but it can be transformed into the modified observer form.

If we let

$$\alpha_1(\xi_2) = \xi_2 - \frac{(\xi_2^0)^2}{2\xi_2^0 - \overline{\xi_2}} \quad ; \quad \overline{\xi_2} = \xi_2^0\left(2 - \frac{\xi_2^0}{\xi_2}\right)$$

$$\alpha_2(\xi_2) = (\xi_2^0)^2\left(k_\rho \ln\left(\frac{\xi_2}{\xi_2^0}\right) + \frac{g}{(\xi_2)^2} - \frac{g}{(\xi_2^0)^2}\right)$$

$$\alpha_3(\xi_2) = \frac{g(\xi_2^0)^2}{k_\rho \xi_2} \tag{22}$$

and

$$x_1 = \xi_1$$

$$x_2 = \xi_2$$

$$x_3 = \frac{\xi_2^0}{\xi_2}\xi_3 + \alpha_2(\xi_2) \tag{23}$$

then the system can be written in modified observer form:

$$\begin{aligned} \dot{y} &= x_1 \\ \dot{x}_1 &= x_2 - \alpha_1(\xi_2) \\ \dot{x}_2 &= x_3 - \alpha_2(\xi_2) \\ \dot{x}_3 &= -\alpha_3(\xi_2) \end{aligned} \tag{24}$$

In some sense this change of coordinates linearizes the system "as much as possible".

Assuming the model to be affected by random process and measurement noises, the corresponding stochastic differential equations become:

$$\begin{aligned} dy &= x_1 dt + D dv \\ dx_1 &= (x_2 - \alpha_1(\xi_2))dt + B_1 dw_1 \\ dx_2 &= (x_3 - \alpha_2(\xi_2))dt + B_2 dw_2 \\ dx_3 &= -\alpha_3(\xi_2)dt + B_3 dw_3 \end{aligned} \tag{25}$$

with

$$x_1(0) = N(x_1^0, P(1,1))$$
$$x_2(0) = N(x_2^0, P(2,2))$$
$$x_3(0) = N(x_3^0, P(3,3))$$
$$Q = BB^T, \; R = DD^T$$

where R is the measurement noise covariance and Q(1,1), Q(2,2), Q(3,3) are the process noise variances. P(1,1), P(2,2), P(3,3) are the variances of the errors in the initial conditions.

The filter dynamic equations are:

$$\begin{aligned} d\hat{y} &= \hat{x}_1 dt \\ d\hat{x}_1 &= (\hat{x}_2 - \alpha_1(\hat{\xi}_2))dt + K_1(dy - d\hat{y}) \\ d\hat{x}_2 &= (\hat{x}_3 - \alpha_2(\hat{\xi}_2))dt + K_2(dy - d\hat{y}) \\ d\hat{x}_3 &= -\alpha_3(\hat{\xi}_2)dt + K_3(dy - d\hat{y}) \end{aligned} \tag{26}$$

Rewriting the system in the original coordinates, the filter equations become:

$$\begin{aligned} d\hat{\zeta}_1 &= \hat{\zeta}_2 dt + K_1(dy - \hat{\zeta}_1 dt) \\ d\hat{\zeta}_2 &= (-g + \rho(\hat{\zeta}_1)\hat{\zeta}_2^2\hat{\zeta}_3)dt + \left(\frac{\hat{\zeta}_2}{\hat{\zeta}_2^0}\right)^2 K_2(dy - \hat{\zeta}_1 dt) \\ d\hat{\zeta}_3 &= \left(\frac{\hat{\zeta}_3}{k_\rho}K_1 - \frac{\hat{\zeta}_2}{k_\rho \rho(\hat{\zeta}_1)(\hat{\zeta}_2^0)^2}K_2 + \frac{1}{\rho(\hat{\zeta}_1)(\hat{\zeta}_2^0)^2}K_3\right)(dy - \hat{\zeta}_1 dt) \end{aligned} \tag{27}$$

where K_1, K_2, K_3, are the gains computed as $K=\bar{P}C^TR^{-1}$ and $\bar{P}$ is obtained from (14).

The Jacobian of the transformation from physical coordinates to observer form is:

$$J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \left(\frac{\zeta_2}{\zeta_2^0}\right)^2 & 0 \\ \frac{\zeta_3}{k_\rho} & \frac{-\zeta_2}{k_\rho \rho(\zeta_2^0)^2} & \frac{1}{\rho(\zeta_2^0)^2} \end{bmatrix} \qquad \rho = \rho_0 e^{-\zeta_1/k_\rho} \tag{28}$$

The Jacobian turns out to be close to a unitary operator. This is clear for the 2x2 upper left minor. In fact, simulations were computed both assuming the Jacobian constant evaluated at ζ^0, and computing its actual value on line. The results were not affected in an appreciable manner.

3 - Simulations and Results

In this section we present the results of some simulations, run for different noise regimes and with the following constants:

initial conditions:

- initial height = 30,000 m
- initial velocity = - 2,000 m/sec
- initial inverse ballistic coefficient = $1.025*10^{-4}$ m^2/kg.

physical parameters:

- atmospheric decay constant $k_\rho = 10{,}000$ m
- atmospheric density at sea level $\rho_0 = 1.230$ kg/m^3
- gravity acceleration $g = 9.81$ m / sec^2

Additionally, the results correspond to the following noise regime:

measurement noise

$R = 100$ $m^2 sec$

process noises

- $Q(1,1) = 100$ m^2
- $Q(2,2) = 100$ m^2/sec^2
- $Q(3,3) = 1.0*10^{-10}$ m^4 / kg^2

uncertainties on the initial conditions

- $P(1,1) = 5000$ m^2
- $P(2,2) = 2000$ m^2/ sec^2
- $P(3,3) = 1.0*10^{-8}$ m^4/ kg^2

The object falls from an initial height of 30,000 m with an initial downward speed of 2,000 m/sec. It has an initial ballistic coefficient of 9,756 kg/m^2, which is equivalent to the object weighing approximately ten metric tons per square meter of surface perpendicular to the direction of the fall.

The errors in the initial conditions are, probably, not unrealistic for a radar tracking problem. The process noise covariances are also close to reality if we consider the possible random effects of changes in the conditions of the atmosphere and wind encountered on the way to the ground along the trajectory. Finally, the process noise on the ballistic coefficient could be interpreted as variations of the shape or orientation of the object during the fall.

The behavior of the real system is presented in Fig. 1, which portrays a typical trajectory of the three states affected by the noises. Note that the first state is the height of the object, the second the velocity and the third the inverse ballistic coefficient.

In Fig. 2(a) are shown the errors between the estimates of the height of the two filters and the height of the real system (Fig. 1(a)). The EKF and the GANF, compared in terms of performance in the estimate of the height, are nearly equivalent. The EKF performed slightly better, but the time history of the error is nearly identical. Similarly, in Figs. 2(b) and 2(c) are shown the estimates of the velocity and the inverse ballistic coefficient, respectively. Although the two filters showed a very similar behavior, the EKF performed slightly better, the difference being mainly due to the approximation introduced in assuming that the noise covariances were constant in observer coordinates for the GANF. In Fig. 2(d) is shown the behavior of the real inverse ballistic coefficient and of its estimates to give the reader an idea of the initial error in the estimates and of the effect of the process noise. Without process noise the real coefficient would be constant.

Shown in Fig. 3(a) is the logarithm of the average covariance of the error in the estimates of the height for 25 Monte Carlo runs. As can be seen from the Figs. 3(a) and 2(a) the recovery from errors in the initial guess is very fast after which the covariance settles down to values close to process noise covariance for the height. There is no appreciable difference in the behavior of the two filters. In Figs. 3(b) and 3(c) are shown the logarithms of the average covariance, for 25 Monte Carlo runs, of the error in the estimates of the velocity and the inverse ballistic coefficient, respectively. Again the two filters performed very similarly. In Fig. 3(c) the two filters behaved so similarly that the two curves are almost indistinguishable. All these results were checked by insuring that the covariances of the errors were near the values of the covariances theoretically predicted by the solution of the Riccati differential equation.

Conclusion:

The simulation results demonstrate that the GANF filter performed practically as well as the extended Kalman filter from the point of view the accuracy of the estimates.

In terms of the algebraic efforts in developing the filter equations, the EKF is obviously more straightforward, since one need only evaluate a first order approximation to the nonlinear equations along the trajectory. The GANF, on the other hand, relies on differential geometric concepts, and require considerably more difficult algebraic computations off-line. However, the development of the algebra may eventually become a simple exercise in computer programming if currently popular symbolic manipulation programs like MACSYMA or SMP are used.

The real advantage of the GANF filter over the extended Kalman filter is in its computational efficiency. In fact, as mentioned previously, we can compute the gains of the GANF filter off-line, whereas the gains of the extended Kalman filter must be calculated on-line. For the particular simulation presented in this paper, written in FORTRAN language and executed on a VAX 785 computer running under the VMS operating system, the integration of the GANF filter along the entire trajectory required 1.06 seconds of CPU time, while the integration of the extended Kalman filter took 3.2

seconds. Thus the GANF filter has performed three times faster. With higher order systems the computational advantage will be further emphasized since the on-line computational burden of the extended Kalman filter grows as $(n^2 + 3n)/2$ while that of the asymptotic nonlinear filter grows only as n. In fact, solving the Riccati differential equation on-line requires the integration of $(n^2 + n)/2$ scalar differential equations.

List of figures:

Fig. 1 Nominal trajectory of the states of the falling object in the presence of random noise. (a): height, (b): velocity, (c): inverse ballistic coefficient.

Fig. 2 Errors of the extended Kalman filter and of the geometric asymptotic nonlinear filter in the estimates of the states of the falling object. (a): height, (b): velocity, (c): inverse ballistic coefficient, (d): estimates of the inverse ballistic coefficient together with the actual inverse ballistic coefficient.

Fig. 3 Variances of the errors of the extended Kalman filter and of the geometric asymptotic nonlinear filter in the estimates of the states of the falling object after 25 Monte Carlo runs. Plotted using logarithmic scale. (a): height, (b): velocity, (c): inverse ballistic coefficient.

References:

[1] Krener, A.J., and A. Isidori, "Linearization by Output Injection and Nonlinear Observers", Systems and Control Letters 3 (1983), pp. 47-52.

[2] Krener, A.J., and W. Respondek, "Nonlinear Observers with Linearizable Error Dynamics", SIAM J. Control and Optimization 23(2) (1985), pp. 197-216.

[3] Gelb, A. (ed.), *Applied Optimal Estimation* (MIT Press, Cambridge, 1974).

[4] Wishner, R.P., J.A. Tabaczynski, and M. Athans, "A Comparison of Three Non-Linear Filters", Automatica 5 (4) (1969), pp. 487-496.

[5] Krener, A.J., "The Asymptotic Approximation of Nonlinear Filters by Linear Filters", in C.I. Byrnes and A. Lindquist, editors, *Theory and Application of Nonlinear Control Systems*, Elsevier Science Publishers B. V. (North Holland) (1986), pp. 359-378.

[6] H. Keller, "Nonlinear Observer Design by Transformation into a Generalized Observer Canonical Form", University of Karlsruhe, preprint.

[7] H. Keller, "Nonlinear Observer Design via Two Canonical Forms", University of Karlsruhe, preprint.

[8] Bestle, D. and M. Zeitz, "Canonical Form Observer Design for Non-linear Time-variable Systems", International Journal of Control, 38(2) (1983), pp. 419-431

[9] Fritz, H. and H. Keller, "Design of Nonlinear Observers by a Two-step

Transformation", in M. Fliess and M. Hazewinkel, editors, *Algebraic and Geometric Methods in Nonlinear Control Theory*, D. Riedel, Dordrecht, 1986.

[10] Li, C.W. and L.W. Tao, "Observing Non-linear Time-variable Systems Through a Canonical Form Observer", International Journal of Control, 44(6) (1986), pp. 1703-1713.

[11] Zeitz, M., "Canonical Forms for Nonlinear Systems", in *Proceedings of the Conference on the Geometric Theory of Nonlinear Control Systems*, Wroclaw Technical University Press, Wroclaw, 1985.

[12] Zeitz, M., "The Extended Luenberger Observer for Nonlinear Systems", preprint.

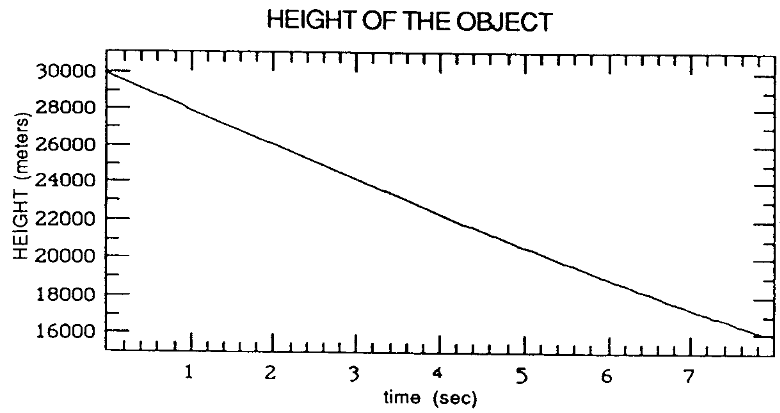

Fig.1.a.

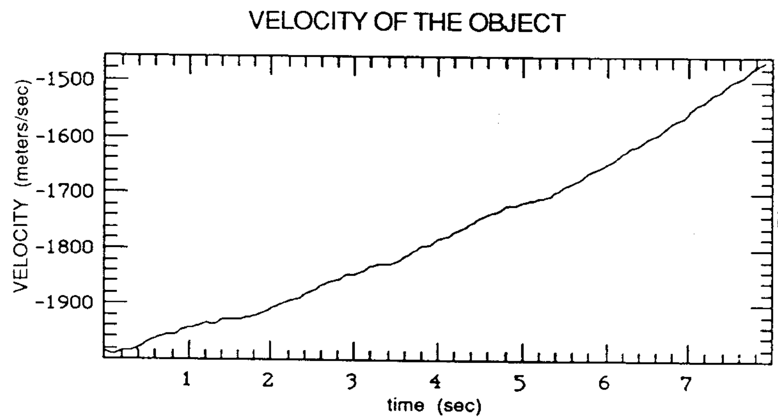

Fig.1.b.

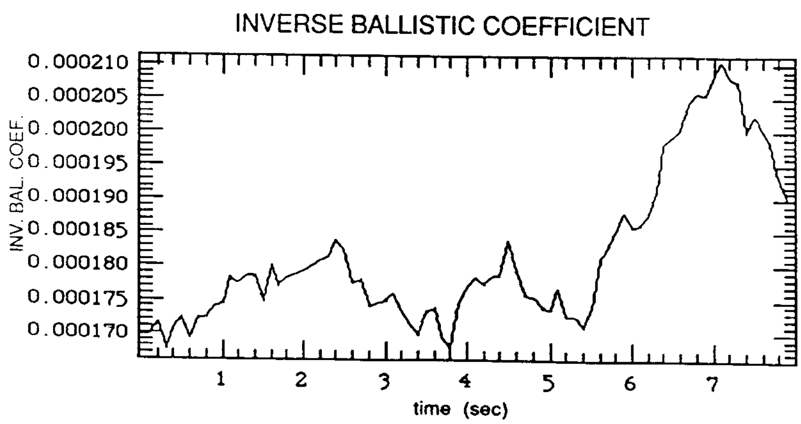

Fig.1.c.

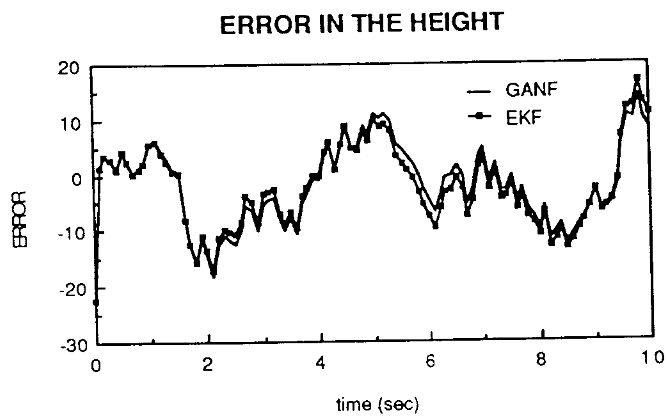

Fig. 2.a.

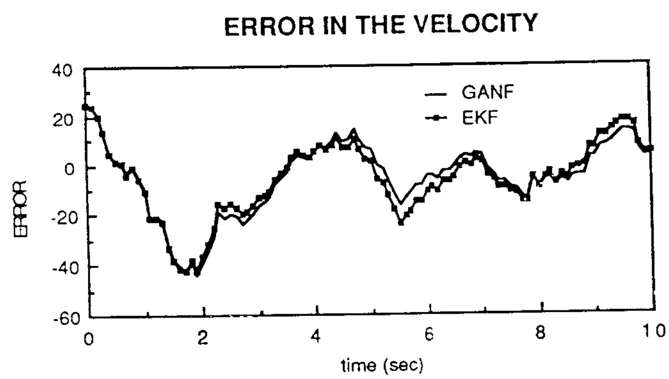

Fig. 2.b.

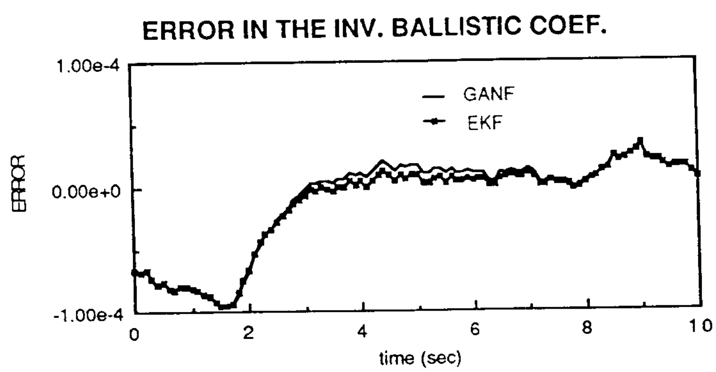

Fig. 2.c

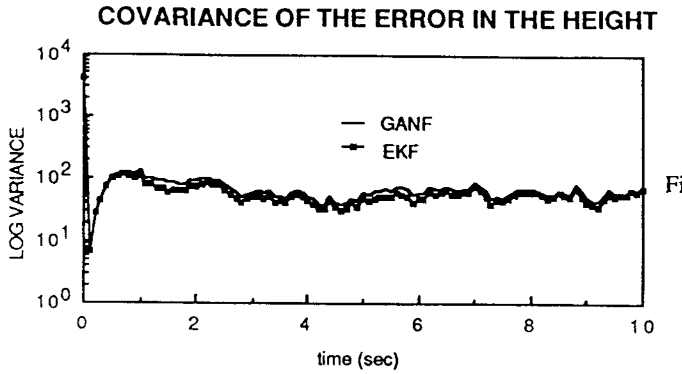

Fig. 3.a.

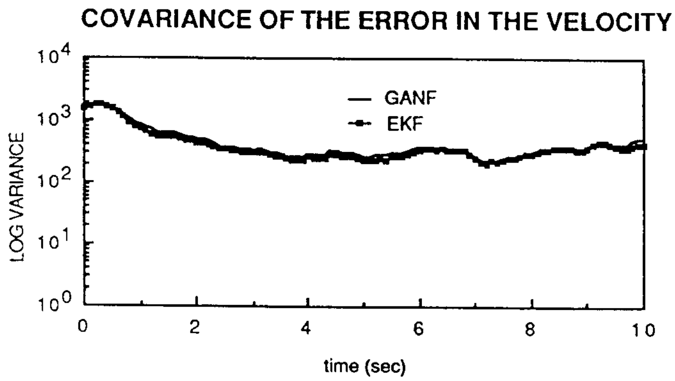

Fig. 3.b.

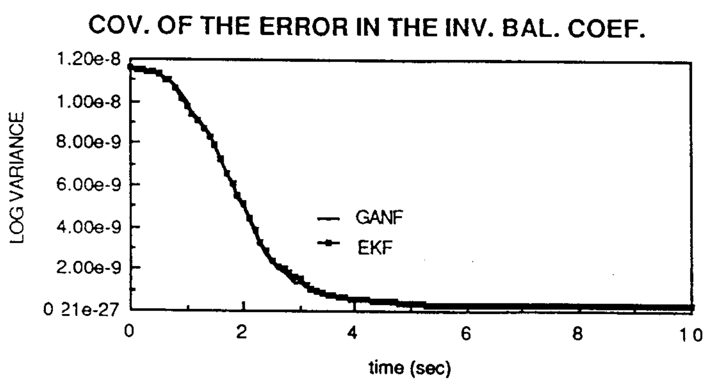

Fig. 3.c.

Fig. 2.a.

Fig. 2.b.

Fig. 2.c