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BOOK REVIEWS

Edited by C. W. Groetsch and K. R. Meyer

Pearls in Graph Theory: A Comprehensive Introduction. *By Nora Hartsfield and Gerhard Ringel.* Academic Press, Boston, 1994. $39.95. 249 pp., hardcover. ISBN 0-12-329553-4.

A "pearl" of mathematics is something of beauty and value—a theorem, a proof, a problem, or just an example—that delights, inspires, or surprises. The field of graph theory is replete with pearls of great variety, and Hartsfield and Ringel have collected many in this wonderful book. (This wide variety suggests that *pearl* may not be the most descriptive word; *jewel* or *gem* might be better, but we will not argue that point.)

Since *Pearls* was written to be a textbook for a course in graph theory, let me begin by reviewing briefly the case for such a course:

- graphs combine the concrete with the abstract in a way that appeals to students;
- graphs are interesting both in their own right and as mathematical models;
- graph theory allows students to discover results and to learn a variety of proof techniques;
- graph theory is at the heart of modern applied mathematics: operations research, algorithms, and complexity theory;
- graph theory has important connections with other areas of mathematics.

Thus graph theory provides a superb way for students to learn mathematics as they have never done before, through discovery and proofs. It can serve both as a transition to higher mathematics and as an end in itself, and can be offered at various levels.

Based on their experience, Hartsfield and Ringel advocate a course requiring little background—only strong high school preparation and an interest in mathematics. This gem-filled book was written for such a course.

Pearls contains a good selection of topics, chosen not only for their level but to meet the discovering-and-proving goals. Here is a brief list of the contents of the ten chapters: basic concepts, colorings, circuits, extremal problems, counting, edge-labelings, applications and algorithms, drawings, measures of nonplanarity, and embeddings.

Themes that prevail are colorings, decompositions, and especially embeddings, reflecting the authors' own research interests. The last chapter, on embeddings, is at a more advanced level than the others. In fact, it would be a good starting place for anyone wanting to study graphs on surfaces in depth.

The level of the book requires that some of the finest pearls of graph theory be omitted; for instance, there is nothing on matrices or automorphism groups. Additionally, some topics ideal for this level are also missing; tournaments, connectivity, and line graphs come to mind.

For many readers of this publication, the chapter on applications and algorithms will be of particular interest. These topics do not feature prominently in the book, but this chapter provides a good introduction. The three algorithms presented are for finding minimum connectors, maximum matchings, and prefix codes. While these provide nice variety, again, some real gems are missing, including an algorithm for finding shortest paths.

The exercises are another strong feature of the book. There is a wide variety of problems, some asking only for straightforward computation of parameters, some asking for proofs (at various levels), and some being exploratory in nature. Even for courses for which the book is at too elementary a level, instructors will find the exercises a valuable resource.

In summary, for a course at an elementary level, this book has much to offer. Instructors with experience in graph theory can substitute and supplement it to meet their needs, while instructors with little or no background in graph theory will find *Pearls* both teacher- and student-friendly.

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Publishers are invited to send books for review to Book Reviews Editor, SIAM, 3600 University City Science Center, Philadelphia, PA, 19104-2688.

Adapted Wavelet Analysis from Theory to Software. *By Mladen Victor Wickerhauser.* A. K. Peters, Boston, MA, 1994. $59.95. xii+486 pp., hardcover. ISBN 1-56881-041-5.

Although wavelet analysis is a very young research area of mathematics, it is perhaps the richest and most rapidly developing subject in the field of applied and computational analysis. Although there are already several popular monographs in the literature devoted to this subject, the book under review is the first one that goes beyond the mathematical treatment to aid those who write computer programs to analyze real data. It addresses the important properties of the wavelet transform so as to establish the criteria by which the proper analysis tool may be chosen, and it then details the software implementations for computational needs. On the other hand, this book is rather self-contained, including even the necessary preliminary materials, such as mathematical analysis in Chapter 1, programming techniques in Chapter 2, and the discrete Fourier transform in Chapter 3. Chapters 4-10 are devoted to the algorithmic approach of wavelet analysis, and the final chapter includes applications to image compression, speech signal segmentation and scrambling, and signal de-noising. In addition, an extensive appendix, giving solutions of selected problems in Chapters 2 and 4-9, as well as several tables of filter coefficients, is included.

The presentation of wavelet analysis in Chapters 4-10 is different from those in the existing wavelet books, such as [1, 2, 3]. Since the discrete Fourier transform has already been reviewed in Chapter 3, the subject of localized trigonometric series (or local trigonometric transforms) presented in Chapter 4 provides a continuous flow of ideas from global to local analyses. Also, since the main concern of this book is computer implementation, a thorough discussion of quadrature mirror filters in subband coding theory in Chapter 5 is probably the most natural approach for introducing the so-called discrete wavelet transform, DWT (or wavelet series), in Chapter 6. Beyond DWT, but still within the realm of discrete computational analysis, are wavelet packets, their corresponding best basis algorithm, and multidimensional library trees, discussed in Chapters 7, 8, and 9, respectively. Of course, a chapter on time-frequency analysis must be included in any book on wavelet applications, and this is done in Chapter 10.

Included in each chapter is a brief discussion of the technicalities of implementation. In addition, examples in pseudocode are given, and a computer diskette in Standard C can be purchased separately. This book, beautifully written by an expert in the field, should be a valuable addition to the personal library for those who are particularly interested in the applications of wavelets to data analysis and signal processing.

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The Theory of the Chemostat: Dynamics of Microbial Competition. *By Hal L. Smith and Paul Waltman.* Cambridge University Press, Cambridge, UK, 1995. $59.95. xvi+313 pp., hardcover. ISBN 0-521-47027-7.

The theory of the chemostat is a genuine success story in mathematical biology. This story involves interdisciplinary cooperation between biologists and mathematicians, mutual feedback between theory and experiment, and an interplay among model derivation, mathematical analysis, biological facts, and laboratory realities. It involves nontrivial and sophisticated mathematics as well as the creation of new methods and concepts. The mathematical results have made testable predictions and stimulated laboratory experiments. The predictive successes of the theory are impressive (especially given the notorious lack of predictability of mathematical models in population dynamics and ecology). The applicability of the laboratory chemostat and its models to other important systems makes the theory relevant to a broad range of practical circumstances, ranging from the ecology of alpine lakes to waste management treatment facilities to the laboratory culturing of generically engineered organisms. For this reason the theory of the chemostat also provides a very fruitful theoretical means for studying general ecological interactions. There are shortcomings in chemostat models, of course, as there are for any mathematical model. These shortcomings, however, serve to further stimulate the interplay between biological modeling and mathematical analysis. As a result, the theory of the chemostat will continue to be a vibrant, active, and important discipline in mathematical biology for a long time to come.

The book by Smith and Waltman, two of the leading authorities on the theory of the chemostat, provides an outstanding introduction to this theory. Starting from the simplest models of the well-stirred chemostat, they lead the reader from the basic theory, through many of its elaborations, up to the frontiers of current research and many open questions and unsolved problems. The first chapter defines the chemostat, its role as an experimental device, and the simplest mathematical models for the population dynamics of both a single microbial organism and two interacting species. The focus is on species competing for a resource designed by the experimenter to be in limited supply (while all other required nutrients are in abundance). The resulting (exploitative) competition for the limiting nutrient is the foundation on which the theory of the chemostat is built. By means of modern methods of nonlinear dynamic systems theory a global description of the asymptotic dynamics can be obtained. Beyond being a beautiful theory the mathematical results provide testable predictions of the competitive exclusion principle which, unlike the famous classical competition model of Lotka and Volterra, successfully predicts the outcome from species-specific parameters measurable in isolation before the interaction takes place. In the second chapter the authors show that these basic global results remain valid for an arbitrary number of species and for general uptake rate functions. The subsequent chapters are organized around the question of to what extent these fundamental results and predictions of the basic chemostat model remain valid under more complicated circumstances or when more complicated phenomena are taken into account. Chapters are included on interactions with more trophic levels, spatial heterogeneity (gradostats), periodic washout rates, variable yield models, and size-structured species. A chapter on "new directions" includes models with delays and diffusion (the unstirred chemostat). Each chapter provides a biological motivation for its main theme.

A great deal of sophisticated mathematics is brought to bear during the course of the book. The book may be read and used in many ways, however. The early chapters are potentially usable in undergraduate courses. Later chapters provide abundant material for researchers interested in contributing to the field. For those readers interested primarily in the biological punch lines, abundant guidelines are given by the authors for which material is key and which may be skipped. At the other extreme, details are given for more mathematically inclined readers (definitions, theorems, proofs, etc.). The book is remarkably self-contained, even for readers with limited backgrounds in dynamic systems. Indeed, the authors do such a good job at writing their material that, on one level, the book could be read as an introduction to many topics in dynamic systems (stability theory, Liapunov theory, Poincaré-Bendixson theory, elementary bifurcation theory, Floquet theory, and more). An appendix supplies a synopsis of many relevant and more advanced topics, including persistence theory, monotone systems, and Leray-Schauder degree theory.

This well-written, well-organized book is destined to become a classic in mathematical biology.

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Polynomial and Matrix Computations Volume 1: Fundamental Algorithms. *By Dario Bini and Victor Pan.* Birkhäuser, Boston, 1994. $64.50. 415 pp., cloth. ISBN 0-8176-3786-9.

This is the first in a planned series of volumes on asymptotically efficient symbolic algorithms for polynomial and matrix computations. Algorithms for "fundamental" problems are presented in the four chapters comprising Volume 1. The first two chapters are concerned with sequential symbolic computations (using exact arithmetic over a ground ring or field and returning exact solutions for problems). Chapter 1 considers polynomial computations, including polynomial evaluation, multiplication, and division; polynomial and rational interpolation and Padé approximation; Chinese remainder computations; computation of minimum spans of linear recurrences; and the computation of greatest common divisors of polynomials. Matrix computations, including the solution of both singular and nonsingular systems of linear equations, and computations of determinants, inverses and generalized inverses, ranks, and characteristic and minimal polynomials over fields, are considered in Chapter 2, both for dense unstructured inputs and for structured inputs (including Toeplitz-like, Cauchy-like, and Vandermonde-like input matrices). Chapter 3 considers Boolean algorithms for integer and finite-precision real computations, including the use of varying (gradually increasing) precision to design algorithms of small Boolean complexity, and to reduce the storage space needed to represent solutions of partial differential equations. Finally, Chapter 4 covers processor-efficient highly parallel symbolic algorithms for the above problems.

Bini and Pan have collected together a substantial amount of material that was previously only available in conference papers and journal articles, including an introduction to asymptotically fast algorithms for polynomial multiplication over arbitrary rings (in Chapter 1), Kaltofen and Pan's processor-efficient parallel algorithms for general matrix inversion over arbitrary fields (in Chapter 4), and the material on computations for dense structured matrices and the varying-precision Boolean algorithms mentioned above. There is also ample new material, including improved algorithms and upper bounds for the sequential and parallel complexities of a number of problems.

Volume 1 also includes an extensive bibliography of work in this area. While there are a few additional references I can think of that might have been included here, these are all either older work (superseded by work that *has* been cited), more appropriate for inclusion in the bibliography of Volume 2, or work describing fast parallel algorithms that are not processor-efficient, and that could be considered to be outside the scope of the text.

I can think of only one algorithm that was known when Volume 1 appeared (indeed, it appears in a paper [7] that is listed in the bibliography but not discussed in the text) that really should have been mentioned in Volume 1, namely, Giesbrecht's efficient algorithm for the computation of the "Frobenius canonical form" of a matrix over an arbitrary field, and its efficient parallel implementation. While canonical forms of matrices are not discussed in Volume 1, computations of characteristic and minimal polynomials of matrices certainly are, and Giesbrecht's algorithm can be used to compute both. To my knowledge, this Las Vegas algorithm (which may report failure with small probability, but always returns a correct answer otherwise) is the first processor-efficient parallel algorithm for computation of the characteristic polynomial of an arbitrary matrix over an arbitrary field. A randomized processor-efficient parallel algorithm for the minimal polynomial had previously been given by Kaltofen and Pan [12], [13], as part of their algorithm for the solution of a system of linear equations. Indeed, Giesbrecht's algorithm uses Kaltofen and Pan's, so Kaltofen and Pan's algorithm will certainly be at least as efficient as Giesbrecht's. However, there is a small positive probability that the output of Kaltofen and Pan's algorithm will be a proper divisor of the minimal polynomial of the input, so that their algorithm is "Monte Carlo" (it can give an incorrect answer without reporting failure). Giesbrecht's algorithm for the computation of the minimal polynomial is, to my knowledge, the only highly parallel algorithm that has been given for this problem that is both "Las Vegas" and processor-efficient.

In fairness, it should be noted that Volume 1 appeared very soon after Giesbrecht's work. Under these circumstances, it is hardly surprising that Giesbrecht's algorithm was overlooked.

Bini and Pan's book contains extremely terse descriptions of a large number of sophisticated algorithms with no examples, very few illustrations, and virtually no white space. I don't know how all this material could have been included in only 400 pages otherwise, but it does make the book quite difficult to read. Thus, while the material on asymptotically fast integer and polynomial arithmetic in Bini and Pan's book is more extensive than the treatment in Aho, Hopcroft, and Ullman [1], I found Aho, Hopcroft, and Ullman's presentation to be somewhat easier to follow. In several cases Bini and Pan's treatment of an algorithm is noticeably terser than the original journal or conference publication; compare, for example, Section 1.9 and [2]. There is also at least one section, Section 2.11 (especially pp. 180–182), where frequent typographical errors complicate matters even more. The original journal articles [8], [9], [10] should be checked before the results mentioned in this section are applied.

The book also contains abundant cross references, including a few more forward references than I would have liked. Once again, I suspect that this was unavoidable, considering the material that Bini and Pan wished to include. However, there are one or two things that could have been done to make the book more readable. An extensive index of definitions and results would have been immensely helpful. Also, results are identified in the text as lemmas, theorems, corollaries, propositions, or facts; all of these are used, and a separate sequence of numbers is used for each. Now, it is not very clear what distinguishes a theorem from a proposition or a fact, so the use of all three of these terms makes it harder to follow the references in the text while accomplishing little else. I hope that Bini and Pan stick to "lemmas," "theorems," and "corollaries" in Volume 2 and, better yet, that a single numbering sequence is used so that, for example, "Lemma 1.1" would be followed by "Theorem 1.2," which would be followed by "Definition 1.3." In the interest of making things easier to find, definitions that appear in the middle of paragraphs of text (such as the definitions of Problems 2.4a and 2.5a on p. 96) should be avoided as well.

There are a few places where careless writing, or failure to note an exception, might cause problems.

Computations over fields of characteristic zero (or even just over subfields of $\mathbb{C}$) are emphasized in much of the book; Sections 1.7 and 4.6 and Remark 4.2.1 discuss techniques to extend many of the results found elsewhere in the book to computations over arbitrary fields, or even arbitrary rings. It might have been helpful, under these circumstances, to say a bit more about those results that cannot be extended easily in this way, such as the results described on pp. 52-53 for "Problem 6.3 (POL DECOMP)." For more information about the decomposition of polynomials over fields of positive characteristic, see von zur Gathen [6].

It is clear that complexity theory is not the subject of this book. Still, I would have preferred that definitions of complexity classes and reductions be more precise and consistent with other work than the ones given here—if definitions were to be included at all. For example, Definition 4.1.2 on p. 299 introduces a complexity class $\mathcal{NC}^k$ without being very specific about the model of parallel computation that is being considered, which might suggest that this complexity class is more robust than it really is. The complexity classes $\mathcal{P}$ and $\mathcal{NC}$ are considered to include search problems (on the following page), and a number of these are identified as being "$\mathcal{P}$-complete." It might have been better to give brief descriptions of these terms without using the "Definition" heading used elsewhere for more precise technical definitions, and to cite work where precise definitions can be found, such as Greenlaw, Hoover, and Ruzzo's book [11] for parallel Boolean complexity theory, or von zur Gathen's article [5] for parallel arithmetic complexity theory.

On p. 42 of Section 1.5, Problems 1.5.1c (SEV POL GCD)—computation of the greatest common divisor of several polynomials—and 1.5.1d (SEV POL LCM)—computation of the least common multiple of several polynomials—are both introduced. An efficient algorithm is given as a solution of the first problem, and it is stated that "this problem, its solution and the complexity estimate can be immediately extended" to the second of these problems. Certainly, a similar algorithm can be used to compute least common multiples. However, it seems unlikely that the "complexity estimate" stated for the GCD problem, $O(mn \log^2 n)$, can be established for the LCM problem as well (and a reader might infer this from Bini and Pan's statement). I find $O(mn \log^2(mn) \log m)$ to be a more plausible bound for the latter problem.

Finally, there are several technical errors in this book. Fortunately, none that I found cause serious problems in the text. In particular, with the exceptions mentioned below, the complexity bounds and algorithms that are given in this book do appear to be correct.

Section 1.8 includes a construction (on p. 61) of an element x of a ring R such that $1, x, x^2, x^3, \ldots, x^{K-1}$ are distinct for a given integer K, which is supposed to work in any ring R with identity that has sufficiently many distinct invertible elements. This construction fails rather badly if R happens to be a direct sum of n copies of the finite field F_3 (so that R contains 2^n invertible elements, but $y^2 = 1$ whenever y belongs to the ring and is invertible) and is reliable only if the multiplicative subgroup of R is cyclic (or if a subring of R has this property, and if sufficiently many invertible elements can be chosen from the subring).

Corollary 1.2, on p. 89 of Section 2.1, asserts that if A is a Hermitian matrix with nonnegative eigenvalues over a field of characteristic zero, then A is "tame;" that is, if A has rank r, then the first r leading minors of A are all nonsingular.

This is simply not true, as one can see by considering the two by two matrix

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

over any field of characteristic zero.

The construction of Section 1.8 is only mentioned twice, at the end of this section and in Remark 4.2.1(c). In both cases, it is applied, correctly, for computations over fields. Corollary 1.2 is the basis for an incorrect deterministic solution of "Problem 2.10b (M PRECONDTN)" given on p. 110. A correct and efficient randomized algorithm is also given for this problem. However, I do not know whether it is possible to solve this problem deterministically, as efficiently as is claimed here. It does not appear that either the corollary or this deterministic algorithm is used elsewhere.

Another error, on pp. 150–151 of Section 1.8, results in an incorrect algorithm for computation of the greatest common divisor of two polynomials. Binary search is used here in order to find the degree of the greatest common divisor of two polynomials (and, therefore, to make Algorithm 8.1 more efficient). The correctness of this improvement seems to be based on the assumption that a linear system (equation (8.3) on p. 149) whose coefficient matrix is a kth subresultant matrix, S_k, is consistent if and only if k is greater than or equal to the degree of the greatest common divisor of the input polynomials.

Suppose, though, that one is computing the greatest common divisor of polynomials $u(x)$ and $v(x)$. Let $u_0(x) = u(x)$, $u_1(x) = v(x)$, $u_2(x)$, $u_3(x), \ldots, u_h(x) = \gcd(u(x), v(x))$, $u_{h+1}(x) = 0$ be the "remainder" sequence of polynomials computed from the inputs using the Euclidean algorithm, and suppose $u_i(x)$ has degree a_i for i between 0 and h, so that if $2 \le i \le h$ then a_i is strictly greater than a_{i+1}. A lemma given, for example, by von zur Gathen [4, Lemma 2.2] implies that system (8.3) is *not* consistent whenever k is greater than or equal to $(a_i + a_{i+1})/2$ and less than a_i, for any $i \le h-1$. Thus, the improved version of Algorithm 8.1 incorporating binary search is only reliable if there are no large drops in degree in the "remainder" sequence of polynomials corresponding to the input, that is, if $a_i = a_{i+1} + 1$ for all i between 2 and $h-1$.

This is easily fixed by modifying equation (8.3) (and the algorithm), essentially as described on pp. 151–152, as part of a solution for "Problem 1.5.1c (SEV POL GCD)." I believe that this will produce a correct algorithm that has the same asymptotic complexity as the incorrect algorithm that appears in the book.

Section 4.6 includes some material on pp. 337–346 that has, apparently, not been published before (based on [14]); this material also has its share of problems.

In the proof given for Proposition 4.6.4, it is not clear whether various block matrices (chiefly, $\pi(V)$ and B) are supposed to have displacement ranks that depend on the field characteristic p (as is suggested at the very beginning of the proof), or whether these should always be equal to two (as seems to be the case after that). I have been informed by one of the authors that this proof is, in fact, incorrect, but that the result can be established by a different approach [15].

Later in this section, on pp. 343–345, a "reduction of Problem 2.2.10 (M RANK) from the case of a Toeplitz-like + Hankel-like input matrix T to the case of a Toeplitz input matrix" is described. Unfortunately, it is assumed here that matrix $V = UTL$ has constant displacement rank, where T is the input matrix, U and L are apparently upper and lower triangular Toeplitz matrices, and displacement ranks are defined in terms of the operator denoted as F_+ on p. 175 (in Section 2.11)—seemingly a requirement that the input matrix T is "Toeplitz-like." The authors have confirmed [3] that this can be fixed, essentially by replacing F_+ in the proof with the operator $F^{\pm}$ from Section 2.11, which characterizes Toeplitz-like + Hankel-like matrices.

Based on all this, I can recommend this book to anyone who intends to do research in this area. I certainly learned a great deal from it. A corrected edition, or one that includes a list of errata, will be even more useful, and I understand that a list of corrections is in preparation. The book might also serve as a reference for a course in symbolic and algebraic computations, either as a supplement for a more elementary text (including the fundamental material in abstract and linear algebra, or perhaps the design and analysis of algorithms) for a senior undergraduate or junior graduate course, or if it is supplemented by conference and journal papers for use in a senior graduate course or seminar series. Given the above technical problems and the level of difficulty of the book, I would not recommend that it be used, in its current form, as the sole reference for a course at any level.

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Lattice Methods for Multiple Integration. *By I. H. Sloan and S. Joe.* Oxford University Press, Oxford, UK, 1995. $69.95. 239 pp., hardcover. ISBN 0-19-853742-8.

Over the past fifteen years there have been major advances in the theory of lattice rules for numerical multiple integration. These rules have become increasingly attractive for practical work because, like Monte-Carlo rules, they consist of simple equally weighted sums of integrand values and they can be easily implemented for high-dimensional integrals. For many integrands, an N-point lattice rule has an error which is asymptotically much smaller than the $O(1/\sqrt{N})$ expected for Monte-Carlo rules. This clearly written book by two of the major contributors in this area is a welcome summary of the most important results in lattice rule theory and application.

The book is written so that it is easily accessible to someone who wants to use lattice rules for practical integration problems or someone who wants a good introduction to lattice rule theory and analysis. Chapter 1 provides a brief introduction to numerical multiple integration. Chapter 2 provides an overview of lattice rules and their most important properties. Chapter 10 has a good discussion of how lattice rules can be efficiently used in practical calculations. Chapter 11 has discussion and results about comparisons with other methods. Appendices A and B provide tables of parameters for lattice rules for integrals with 2-12 dimensions. Someone who wants to learn a little about lattice rules and use them for a practical integration problem should need only an hour or two reading Chapters 1-2, 10-11, and the Appendices, and a little time programming, before some numerical results are available. I would have liked to have seen a more extensive set of tables in the Appendices. Because of their potential for use with high-dimensional integrals, it would have been nice to have tables for good lattice rule parameters for dimension as high as 20. The authors describe methods for finding these parameters, but finding good parameters is often a very computationally intensive process that is not easily carried out by someone who wants to use the rules for a particular practical problem.

Chapters 3–9 focus on a detailed theoretical analysis of lattice rules that includes discussion of different types of lattice rules and derivation of error bounds. These chapters should provide a good summary of lattice rule theory for current and potential contributors in this area. These chapters often begin with a two-dimensional example that is used to introduce and illustrate the rest of the material in the chapter. This should allow someone who is not familiar with this type of material to gradually get used to the terminology and notation. The primary analytical techniques that are used come from Fourier analysis and number theory. Most of the theorems are given with complete proofs, but there is also a fairly extensive set of references given for readers who want further details for some of the background material and quoted results.

In summary, this book should be very useful for a wide range of people interested in lattice rules. It is well written in a style that makes much of the material accessible to statisticians, scientists, and engineers with practical numerical multiple integration problems. There is also a broad foundation of theoretical material for people who want to make further contributions in this area.

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Dirichlet Forms and Symmetric Markov Processes. *By Masatoshi Fukushima, Yoichi Oshima, and Masayoshi Takeda.* Walter de Gruyter, Berlin, 1994. DM168. vii + 392 pp., cloth. ISBN 3-11-011626-X.

This book is a greatly expanded modern version of Fukushima's earlier monograph [F]. Like its predecessor, it consists of two separate but closely related parts. Part I contains the basic theory of symmetric Dirichlet forms with special emphasis on the potential theory of such a form. There is no probability theory in this part. In Part II this nonprobabilistic theory is related to the probabilistic potential theory of a symmetric Markov process.

A Dirichlet form is a closed, densely defined, symmetric form $\mathcal{E}$ on a real L^2-space, $L^2(X, m)$, which satisfies the following Markov condition: $u \in \mathcal{D}(\mathcal{E})$, the domain of $\mathcal{E}$, implies $v := (u \wedge 1) \vee 0 \in \mathcal{D}(\mathcal{E})$ and $\mathcal{E}(v, v) \leq \mathcal{E}(u, u)$. Here (X, m) is a σ-finite measure space. In Chapter 1 the basic relationships among such forms, semigroups, resolvents, and generators are developed. Beginning in Chapter 2 the Dirichlet form is assumed to be *regular*, essentially that $\mathcal{D}(\mathcal{E})$ contains enough continuous functions. Of course, this entails topological conditions on X. The authors assume for the most part that X is a locally compact separable metric space and that m is a Radon measure with full support. Under these assumptions capacity, quasi continuity, equilibrium potentials, and measures of finite energy are introduced and the associated potential theory is developed in some detail. Thus regular Dirichlet forms provide an axiomatic basis for the development of a considerable portion of potential theory. This is an L^2-theory and, at least in this book, a self-adjoint theory. There is a version of this theory in which the symmetry of the form is relaxed, but not completely eliminated. This "nearly" symmetric version is developed in the recent monograph of Ma and Röckner [MR]. However, even this does not cover, for example, the potential theory of the heat equation. On the other hand there is a voluminous literature on various axiomatic approaches to potential theory, many of which apply to the heat equation.

In Part II the authors assume that a (weakly) m-symmetric Hunt process M on X is given. It is easily seen that the transition semigroup $(p_t)_{t>0}$ of M uniquely determines a Dirichlet form $\mathcal{E}$ on $L^2(X, m)$. After some preliminary results it is assumed that $\mathcal{E}$ is, in fact, regular. The various potential theoretic objects from Part I are then interpreted probabilistically. For example, if B is a nearly Borel set of finite capacity and σ_B is its hitting time, then $p_B^1(x) := E_x(e^{-\sigma_B})$ is shown to be a quasi continuous version of the $(1-)$ equilibrium potential e_B introduced in Part I. Chapter 4 is devoted to the interplay between the concepts of Part I and their probabilistic counterparts.

For me the highlight of the book is Chapter 5, the longest chapter, which is devoted to additive functionals. It begins with a characterization of positive continuous additive functionals (PCAF) as those whose Revuz measures are smooth, i.e., do not charge sets of capacity zero and satisfy a certain finiteness condition that is stronger than being σ-finite but weaker than being Radon. (To the best of my knowledge the name "smooth" for this class of measures was first used by McKean and Tanaka in 1961 [MT] in discussing PCAFs of Brownian motion.) Next Fukushima's celebrated decomposition of $A_t = u(X_t) - u(X_0)$ for u in the extended Dirichlet space as the (unique) sum of a martingale additive functional and a continuous additive functional of zero energy is presented. This leads to a probabilistic interpretation of the Beurling-Deny formula. Those u for which the zero energy part in the above decomposition is of bounded variation are characterized. This chapter is a wonderful example of how the two subjects—Dirichlet forms and Markov processes—combine to enrich each other.

Chapter 6 is devoted to various transformations of forms and processes such as perturbations, killing, time changes, and transformations by supermartingale multiplicative functionals. Many of the results in this chapter were discovered after the appearance of [F].

When studying probabilistic potential theory, at some point one must confront the problem of existence. One usually begins with some analytic (i.e., nonprobabilistic) object such as a differential operator, a "potential" operator, a resolvent, a semigroup, a cone of excessive functions, or a Dirichlet form, or perhaps some "preversion" of these objects, and then under appropriate hypotheses constructs a Markov process which corresponds to the given analytic object. A Dirichlet form is a particularly tractable and useful starting point, and in Chapter 7 the authors establish the basic existence theory, namely, that given a regular Dirichlet form $\mathcal{E}$ there exists a (weakly) m-symmetric Hunt process M such that $\mathcal{E}$ is the form determined by M. The use of Dirichlet forms to construct processes has been especially useful on manifolds and infinite-dimensional spaces. Strictly speaking the theorems established in §§7.1 and 7.2 do not apply in infinite-dimensional situations, since the underlying space has been assumed to be locally compact. In §7.3 the authors present an existence theory for a Hunt process (in the restricted sense) on a Lusin space. However, the tightness condition imposed (7.3.2) is very strong. For example, the theorem would not be applicable to Brownian motion killed on leaving the unit ball in Euclidean space. But Theorem 7.3.1 remains essentially true under a weaker tightness condition. See, for example, Theorem IV.3.5 and Section V.2 of [MR] for an extended treatment of the relevant issues.

An important feature of this book is the detailed discussion of numerous nontrivial examples. An index of the examples would have been very useful. Finally, there is an appendix. Among other things it contains an excellent introduction to Hunt processes and to martingale additive functionals over them. The book is very carefully written and is remarkably free of misprints. My one quibble is the numbering system. Theorems, corollaries, lemmas, examples, displays, etc. are all numbered separately. This makes it much more difficult, at least for this reader, to track down specific references as they occur in the text.

This is an important book that will undoubtedly become a standard reference in the field. The authors should be commended for undertaking the formidable task of writing an up-to-date account of this vital subject and congratulated for bringing it off so well.

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Applied Discriminant Analysis. *By Carl J. Huberty*. John Wiley, New York, 1994. $59.95. xxii + 466 pp., cloth. ISBN 0-471-31145-6.

This book is addressed primarily to graduate students, applied researchers, and methodologists interested in the discriminant analysis. To understand the book, it is desirable to be familiar with multiple regression, multiple correlation, parameter estimation, and associated concepts. It has twenty chapters which are divided into four parts.

Part One: Introduction (Chapters I and II). In this part the author discusses the statistical concepts, matrix algebra, and multivariate statistical method.

Part Two: Prediction (Chapters III-XII). In this part methods for assessing the goodness of the classification rule to predict unit group membership are discussed. Some nonnormal rules are also given. Then the problems of predictor selection and predictor ordering are studied.

Part Three: Description (Chapters XIII-XVIII). Discussion in this part pertains to the analysis and description of effects of the grouping variables on the outcome variables. Methods of outcome variable selection and ordering are also described.

Part Four: Issues and Problems (Chapters XIX and XX). In the last two chapters some issues related to statistical analysis and problems associated with the use of discriminant analysis are discussed.

The above twenty chapters are followed by three appendices and a diskette. The three appendices contain the Data Set Descriptions, Computer Printouts, and Content of Accompanying Diskette. Answers to exercises given at the end of some of the chapters are also provided.

The format of the chapters is good: starting with an introduction, giving the methodology to be used, and including numerical examples well worked-out. This is, however, an applied book and no claim to mathematical niceties is made. An outstanding feature of the book is the inclusion of four real data sets which are utilized to illustrate various analysis results, in both examples and exercises. These numerical illustrations are obtained via the BMDP, SAS, and SPSS.

It is noticeable that the author includes 23 pages of references and 137 pages of computer printouts, which comprise 34.33% of the book.

It is a readable book which provides a valuable practical introduction to applied discriminant analysis. Interest in the applications of discriminant analysis has grown rapidly, as attested by the large number of references. Hence, it is a welcome addition.

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The Complex WKB Method for Nonlinear Equations 1: Linear Theory. *By Victor P. Maslov*, translated from Russian by M. A. Shishkova and A. B. Sosinsky. Birkhäuser-Verlag, Basel, 1994. $123.00. vii + 300 pp., hardcover. ISBN 3-7643-5088-1.

This is a long overdue book by one of the great names in the field of asymptotic expansions for partial differential equations. The Russian book on which it is based appeared in 1977, but the translation was long delayed. It has finally appeared, with appendices to bring some material up to date.

What is the WKB method? Physicists most often use the term in the semiclassical (or quasiclassical) problem to unveil the classical mechanics hidden in the Schrödinger equation

$$(1)\qquad i\hbar\,\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\,\nabla^2\psi + V(x,t)\,\psi,$$

as Planck's constant $\hbar$ tends to 0. (While it sounds silly to speak of a constant tending to 0, in macroscopic units, $\hbar$ is about 10^{-27} erg-sec.) The simplest analyses are for the eigenmodes of a one-dimensional version of (1), the ordinary differential equation

$$(2)\qquad -\frac{\hbar^2}{2m}\,\psi'' + V(x)\,\psi = E\psi.$$

The limit as $\hbar \to 0$ is an important and familiar problem in singular perturbation theory. As has been known for a long time, good approximate solutions of (2) can be constructed as linear combinations of

$$(V(x)-E)^{-1/4}\exp\left(\pm\frac{1}{\hbar}\int_{x_0}^{x}\sqrt{V(y)-E}\,dy\right), \qquad (3)$$

where for convenience I set $2m = 1$. Rather obviously, there are difficulties in the vicinity of the turning points, values of x for which $V(x) = E$, but elsewhere the functions (3) form a fundamental set of solutions for an equation differing from (2) by a small term ($O(\hbar^2)$ and no derivatives of ψ). The semiclassical problem is central to quantum mechanics and the idea of using approximate solutions of the form (3) to study (2) is an excellent one. The folk history attributing this idea to Wentzel, Kramers, and Brillouin is, however, all wrong.

The Schrödinger equation was new in 1926, and was used to solve many problems that had been out of the reach of the old quantum theory, which was based on the prescription of Wilson and Sommerfeld that classical closed orbits should be quantized by a condition on the action integral,

$$\oint p\,dq = k\hbar,$$

where k is an integer. The most notable success of the old theory was the prediction of the Balmer spectrum of hydrogen, which depends only on the differences of energies, so that k might as well be replaced by $k+\alpha$ for any constant α. Indeed in some situations physicists had set $\alpha = 1/2$ on an ad hoc basis, so a more general formulation of the Sommerfeld-Wilson quantization rule was

$$\oint p\,dq = (k+\alpha)\,\hbar, \tag{4}$$

with a somewhat mysterious constant α.

It was clearly important to recover the successful calculations of the old quantum theory by a singular perturbation analysis of the Schrödinger equation, and so Brillouin [3, 4] and Wentzel [17] recovered (4) with $\alpha = 0$, using, respectively, implicit and explicit Riccati transformations applied to (2). Kramers [9] used a more sophisticated approach related to the approximations (3)—a Riccati transformation neglects the very useful prefactor at leading order—and obtained (4) with $\alpha = 1/2$, which turns out to be more nearly correct in most, but not all, simple situations. The point, though, is that these approximations were already rather well known and standard since the work of Liouville [10] and Green [5], and the contributions of Wentzel, Kramers, and Brillouin were to cope with the turning points, not to think up the approximations (3). Kramers used solutions of Airy's equation

$$y'' = C(x - x_0)y$$

to interpolate in the vicinity of a linear turning point at x_0, and found that he needed to set $\alpha = 1/2$ in order for the phase of the solution to match properly around a turning point, making the wave function single-valued. This makes α proportional to a winding number, and today, especially because of the influence of Keller and Maslov, we would recognize it as a topological index.

Jeffreys [7], as luck would have it, solved the same problem in the same way as Kramers, but two years earlier, before Schrödinger made it fashionable. His was a good, thorough paper, but poor Jeffreys' initial is only occasionally coupled with W, K, and B for contributions to the turning-point problem, or when credit for the approximation of Liouville and Green is wrongly bestowed elsewhere. Strictly speaking, if there is no turning point, it's not the WKB method, and if there is, it would be more accurate to call it the J method. (But I don't really imagine that this book review can stem the terminological tide in favor of WKB!)

The authoritative history of these methods in the context of ordinary differential equations is [13]. While the level of analysis at the time of which I speak was often formal, satisfactory theorems guaranteeing uniform approximation arrived over the years; for instance, see [15].

There is much more to semiclassical physics than this method for ordinary differential equations. Schrödinger had concocted his equation while thinking about Hamilton-Jacobi theory, and even in the multidimensional, time-dependent setting, the Hamilton-Jacobi equations of classical dynamics emerge from (1) with a careful, small $-\hbar$ expansion. Brillouin [4] noticed this, but the first clear analysis was the remarkable though neglected article of Van Vleck in 1928 [16]. Van Vleck showed that a wave function can be written asymptotically in the form

$$A^{1/2}\Delta^{1/2}\exp(iS/\hbar), \tag{5}$$

where the action integral S satisfies the Hamilton-Jacobi equation of classical mechanics, Δ is a functional determinant of the canonical transformation generated by S, and A is a normalization factor. Van Vleck realized that quantum conditions (4) result from a condition of single-valuedness of the wave function ψ regardless of dimensionality. His factor Δ directly measures the convergence of classical paths, showing that the caustics of the classical motion replace the turning points when the dimension is increased, and determine α in (4). He neglected to calculate α systematically, however, and only much later Keller [8] and then Maslov [11] rediscovered Van Vleck's ideas and rectified this. In their hands it was recognized that the solutions of the Hamilton-Jacobi equation define submanifolds of phase space with a Lagrangian structure, the topology of which can be characterized by integral invariants related to α. Maslov particularly stressed the central importance of these Lagrangian manifolds and introduced canonical operators, now often called "Maslov operators," on them. The understanding of this structure allows one to develop systematic expansions, essentially by summing terms of the form identified by Van Vleck and expanding. (For a clear, brief description of Lagrangian manifolds and the Keller-Maslov index, see Appendices 11 and 12 of [2].) A closely related construction arises in geometric optics, where the eikonal equation plays the rôle for the wave equation that the Hamilton-Jacobi equation plays for the Schrödinger equation. Similar methods apply to many other partial differential equations, especially but not exclusively linear ones.

This is not the first book by Maslov on his methods, and apparently not the last, although volume II, on the nonlinear situation, will not appear in the near future. The earlier books [11] and [12] dealt mainly with quantum mechanics. The present book uses similar methods for more general partial differential equations, while Schrödinger's equation continues to be an important topic. There is also significant overlap with the book on Maslov's methods by Mishchenko, Shatalov, and Sternin [14].

Maslov begins with an asymptotic analysis of equations such as the wave equation in the high-frequency regime or the Klein-Gordon equation. For Maslov a "WKB solution" (let's forgive the terminology since caustics are involved) is of the form (5), which in his notation reads

$$A(1/h)[\varphi_0(x,t)\exp(iS(x,t)/h)+O(h)].$$

Maslov forces S to be a solution, generally complex, of a Hamilton-Jacobi equation

$$\partial S/\partial t + H(\partial S/\partial x, x, t) = 0,$$

while the function φ can be expanded in the small parameter h, and will satisfy a transport equation involving S. The pair of equations for S and φ is called the canonical system. In the first chapter he also introduces a precise notion of asymptotic equivalence of functions, allowing more flexibility than the usual Erdélyi symbols, and relates this equivalence to an inequality of Gårding. Chapter II begins with a model problem and introduces Lagrangian manifolds, first with real and then with complex germs, and discusses their evolution under a Hamiltonian H. Action functions are defined on these manifolds. Construction of approximate solutions of Hamilton-Jacobi and transport equations begins in earnest at the end of this chapter and is the subject of Chapter III. In Chapter IV we learn how to construct eigenfunctions of differential and pseudodifferential operators from S and φ.

There are three appendices with more recent material than in the 1977 Russian edition. Appendix B, for example, deals with the tunneling effect and gives calculations in Maslov's framework for exponential decay asymptotics of the sort done by Helffer and Sjöstrand and Simon in the 1980's (see [6] and references therein). The book concludes with an avowedly heuristic discussion of asymptotics and the saddle point method.

There has long been an air of controversy about Maslov and his methods. Maslov's ideas were recognized early as innovative and important, and his rise to power in Soviet science was swift. Nevertheless, he sometimes met with resistance both in Russia and in the West. There has often been a feeling that his methods were not quite rigorous; this is probably why his latest book appeared in the series *Progress in Physics*, rather than *Progress in Mathematics*, despite having the look and feel of a mathematics text. Maslov has not been known for the clarity of his exposition, which has not helped. When reading other people's major works, even an expert in an intricate area of analysis like asymptotics must choose one of three courses: to invest an enormous amount of time in checking and reconstructing arguments; to understand the main features and trust or doubt the details; or to maintain an attitude of skepticism. Nonmathematical considerations of reputation, personalities, language, and academic politics, and even international politics, can affect our willingness to check or to trust. This writer does not feel in a position to judge the rigor of Maslov's details, but the ideas are compelling, and they emerge fairly clearly in this book, one of Maslov's more readable works. (An unfortunate feature which must be mentioned about readability, however, is the lack of an index.)

History has validated Maslov's ideas. They have been seized upon by Arnol'd [1] and others to build unquestionably rigorous structures, and they have been used by applied mathematicians to forge accurate numerical schemes for problems in physics, oceanography, and other fields. It is truly regrettable that this very significant book did not appear in English many years ago.

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Exponential Attractors for Dissipative Evolution Equations. *By A. Eden, C. Foias, B. Nicolaenko, and R. Témam.* John Wiley, Chichester, UK, 1994. $39.95. viii + 182 pp., paperback. ISBN 0-471-95223-0.

This book develops a relatively new low-dimensional tool for the study of a certain class of dissipative partial differential equations. Such an equation is characterized by the existence of an absorbing set. The ω-limit set of the absorbing set comprises the *global attractor* $\mathcal{A}$, which is compact, has finite fractal dimension, and contains all the long time behavior of the system. It may, however, attract certain solutions at a rate which is only algebraic. Many such equations are also known to possess *inertial manifolds* which are smooth, positively invariant, finite-dimensional manifolds, and which attract all solutions at an exponential rate [1]. The restriction of the flow to such a manifold yields an *inertial form*, a finite system of ordinary differential equations which shares the exact long time behavior of the PDE. This has inspired a number of *approximate inertial manifolds* and related numerical methods.

Notably absent, however, from the list of systems covered by the existing inertial manifold theory are the Navier-Stokes equations (NSE). The difficulty is that a certain spectral gap condition is not satisfied by the Stokes operator, even in the 2D, periodic case.

The alternative theory, emphasized in this book, relaxes the smoothness condition needed for an inertial manifold, and calls the more general object with the remaining properties an *exponential attractor*. The advantage is that this new object can be constructed under less restrictive assumptions, in particular for the NSE. Another advantage is that the dimension of an exponential attractor can be shown to be much closer to that of the global attractor than in the case of the inertial manifold. In some sense exponential attractors are more robust than global attractors. The disadvantage is that the dynamics on this set are generated by a system of ODEs which are much harder to study and approximate.

The theory of exponential attractors is first developed for a discrete dynamical system described by a Lipschitz map S. The main assumption is that S satisfies a weak form of the *squeezing property*. This stipulates that given a contraction rate, there is a finite-dimensional orthogonal projector such that any two points in phase space have images which are either contracted at the given rate or form a difference vector with projected component dominating the complement.

The key mechanism involves a compact invariant set X and the image Z under S of the intersection of X with a ball of some radius R. The projector is injective on the maximal subset $E \subset Z$ on which the latter condition of the squeezing property holds. Thus finite covers of the projection of E lift to all of E. This is used to estimate the number of balls of radius $\rho = \theta R$ needed to cover E in terms of ρ and the dimension of the projector.

The actual construction of the exponential attractor is then carried out by recursively generating covers as above for collections of maximal sets $E^{(k)}$ within refined coverings of $S^k(X)$, with $X = \mathcal{A}$. To control the fractal dimension, the same reduction factor θ and hence the same number of balls are used at each stage on each intersection of $S^{k-1}(X)$ with the previous collection of balls. The exponential attractor is then defined as

$$\mathcal{M} = \mathcal{A} \cup \left(\cup_{j=0}^{\infty} \cup_{k=1}^{\infty} S^j(E^{(k)}) \right).$$

The initial ball is taken large enough to contain all of $X \supset \mathcal{A}$. Note that iteration of the mapping S is actually carried out twice: once in the generation of the sets $E^{(k)}$ and then again over the index j.

This amounts to what the authors appropriately term a "fractal expansion" of the attractor. Indeed, a sharp estimate for the fractal dimension of the global attractor follows easily from this construction.

The construction of exponential attractors is then extended to the continuous flow of a general evolution equation. Specific applications are made to the Kuramoto-Sivashinsky equation, the Kolmogorov-Spiegel-Sivashinsky equation, the Burgers equation, certain reaction diffusion equations, and, of course, the 2D NSE. A special treatment is provided as well for second-order equations including the sine-Gordon and Klein-Gordon equations. The robustness of the exponential attractor is established in the particular case of Galerkin approximations. For completeness, a chapter on inertial manifolds along with an appendix with essentials of dimension theory is included.

Most intriguing, however, is the development of a new approach to the dynamics on the exponential attractor, which sidesteps the nonregular nature of this set. The main result is that, given an exponential attractor for a finite-dimensional evolution equation, there exists a *low*-dimensional generalized dynamical system with the same exponential attractor and the same dynamics on that set. A similar result is also shown for the case where the exponential attractor is replaced by the global attractor. The requirement that the original equation must be finite-dimensional still allows for a broad application to PDEs. In particular, it applies to all equations possessing inertial manifolds, and more generally to Galerkin approximations of those in the wider class of equations having exponential attractors. The latter case suggests that if we can understand the behavior of low-dimensional dynamical systems, we can explain what is generated by arbitrarily fine discretizations of such equations.

The book provides a very reasonable entry into the general low-dimensional approach to PDEs. While not written in quite the introductory fashion of [2] or [3], it is essentially self-contained. This book presents genuinely new results, yet is not as dense as [4], which was devoted solely to inertial manifolds. One could certainly explore the material in a topics course, using the book reviewed here. There is even a brief but provocative list of open questions at the end. Of course, the book is a must for anyone working in the area.

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Theory of Chattering Control. *By M. I. Zelikin and V. F. Borisov.* Birkhäuser, Boston, MA, 1994. $74.50. 242 pp., cloth. ISBN 0-8176-3618-8.

This book deals with a little known but important area of optimal control. One might reasonably expect that a nice optimal control problem should have a nice solution. When the Pontryagin Maximum Principle was first announced, it was sometimes felt that the assumption of measurable controls was only a technical device, as it was believed that most if not all reasonable optimal control problems have smooth or piecewise smooth solutions. In 1961, A. T. Fuller [F] put this myth to rest by exhibiting a very simple optimal control problem whose solution has an infinite number of discontinuities in a finite interval. The control is required to lie in a bounded interval, and the optimal solution switches back and forth (chatters) between its bounds. The interswitch times form a summable geometric series so that an infinite number of switches occurs in a finite time. Fuller's example is hardly pathological; it goes from one position and velocity to another while minimizing the square integral of the position using the acceleration as a control. The control is required to lie between ± 1. If instead the velocity is the control then there is no chatter; if the control is the acceleration or a higher time derivative of the acceleration, then there can be chatter depending on the initial and final conditions.

Many optimal control problems have a dual optimal estimation problem; for example, the dual of the linear-quadratic regulator is the Kalman-Bucy filter. The dual of Fuller's problem was found by D. E. Johansen [J] and Berkovitz and Pollard [B-P]. It finds the minimax least squares linear estimator of the one-dimensional position of an object whose acceleration is bounded by ± 1 from measurements corrupted by white Gaussian noise. The weighing pattern of the optimal filter chatters between positive and negative values like Fuller's optimal control. This topic is not discussed in the present book.

There has been continuing interest in Fuller-like chattering, as it is one of the foremost obstacles to our understanding of regular synthesis (in the sense of Boltyanskii) of optimal control problems. Any general theory of nonlinear optimal control that includes control constraints must deal with chattering.

The present book is an excellent exposition of the current state of knowledge about chattering. It begins in Chapter 1 with a brief introduction to Hamiltonian systems before introducing Fuller's Problem in Chapter 2. The third chapter is the heart of the book, describing various aspects of chattering. The fourth chapter is a readable exposition of the work of I. Kupka, who showed that chattering is ubiquitous in eight- or more dimensional problems. The last three chapters deal with higher-dimensional systems and applications.

Despite the technical nature of the subject matter, the book is well written, self-contained, and relatively easy to read. Until the chattering phenomenon is fully tamed, it will be an obstacle to any complete theory of nonlinear optimal control. This book is an excellent introduction to and survey of chattering optimal controls.

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Fuzzy Logic: A Practical Approach. *By M. McNeill and E. Thro.* Forward by R. R. Yager. AP Professional, Chestnut Hill, MA, 1995. $39.95. 292 pp., hardcover. ISBN 0-12485-965-8.

What is the intended audience? The authors do not specify any audience. Is it for mathematicians? Certainly not. Perhaps the authors believed that by introducing the matter of a hypercube, out of context, and by mentioning a couple of definitions of distances for metric spaces it would make theirs a math text. In this they failed. Is it for engineers? No. Is it for computer scientists? I am not sure. There are a lot of detailed programs in Appendix D, but then again, there is more to computer science than writing programs. The book seems to have been written with the objective of convincing the reader that there exists fuzziness as a new source of uncertainty. In this sense, the writers succeeded splendidly.

There are problems with the exposition also. The narrative is often broken (or disrupted) by remarks of various kinds. These are described as *e-mail from Dr. Fuzzy*. Although they seem to intend to convey a funny interruption or some form of clarification, they actually introduce very fuzzy interjections which do not help the discourse at all.

What has taken a long time to figure out is the reason for "practical approach" in the title. What is practical? The concept of fuzziness or their programming? In this sense, the authors clearly promote the use of the accompanying diskette. There is no explanation anywhere of how the canned programs work—not even a definition of fuzzy set or fuzzy number is included. The tables intended to clarify fuzzy operations, in fact, cloud the issue in puzzling details. It is doubtful that anyone who is totally unfamiliar with the subject of fuzziness will know much that is worthwhile after going through this book.

The text begins with a very old example of the verbal description of the procedure of parallel parking. It continues with six chapters of various items loosely connected to the core of the subject, and it concludes with five additional appendices, the first on fuzzy associative memories and the next on fuzzy sets as hypercube points. This is unnecessary because Zadeh very effectively presented them as sets with a boundary that is difficult to define sharply. It is a very good way of confirming the purpose of such sets, namely, of being useful whenever the concept of belonging is not crystal clear and therefore too complex to represent in a binary mode. We have a lengthy description of disk files. The last two sections are on, respectively, inference engine programs and other fuzzy architecture. The bibliography is rather skimpy. The section on articles and books is very disappointing. Additionally, the section on proceedings does not mention the annual meetings of the North American Fuzzy Information Processing Society (NAFIPS) or of the International Fuzzy Systems Association (IFSA) which holds meetings every other year.

After several years of total neglect and of downright hostility, the American scientific community has realized how modern technology has profited from pursuing the concept of fuzziness and now there seems to be a scramble to catch up. There are several texts in print or to appear soon that do justice to the topic as a very worthwhile field of research. Unfortunately, this is *not* one of them.

In conclusion, this book is probably a prototype of many others to follow now that the concept of fuzziness has finally caught the attention of the American scientific community. It is not to be recommended in any other sense as easy reading—unfortunately with the result of trivializing the topic.

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Fuzzy Logic with Engineering Applications. *By Timothy Ross.* McGraw-Hill, New York, 1995. $59.75. 600 pp., hardcover. ISBN 0-07053-917-0.

Fuzziness is a new science that belongs within the realm of uncertainty. It deals with uncertainty that cannot be easily weighed by numerical quantification because it is imponderable and because it permeates every aspect of our lives. Particularly in the area of modern technology referred to as intelligent systems, the role of fuzziness has become central.

There are two notable periods in its development. First, in 1965 the seminal paper by Lotfi Zadeh brought forth the topic of the fuzzy sets, namely, sets which have undefinable boundaries. This idea was scoffed at for a long time by the mathematical community. It was not until a decade or so ago that, with the extraordinary success of the application of fuzzy logic to control programs by Japanese researchers, the topic of fuzzy logic caught the attention of the engineering community in the USA. Since then, a great deal of attention has been devoted to the topic and quite a few books have been written. Many of them leave a lot to be desired.

This book is an exception. Written by an engineer, for engineers, its exposition is clear, lively, and certainly holds the reader's attention. For example, it has one clear statement on p. 13 of the distinction between fuzziness and randomness: "Fuzziness describes the ambiguity of an event whereas randomness describes the uncertainty in the occurrence of the event." It also proposes a careful treatment of the analysis of fuzzy logic, not only the applications. It is, therefore, a delightful surprise among the many recent books dealing with the application of fuzzy logic to engineering. Its content is the usual: it begins with the introduction, in which the author discusses classical sets and fuzzy sets, then classical relations and fuzzy relations. It continues with membership functions, fuzzy-to-crisp conversion, and fuzzy arithmetic. This chapter includes the discussion of numbers, vectors, and the extension principle. The author then proceeds to classical logic and fuzzy logic discussion on fuzzy rule-based systems. Then he includes fuzzy nonlinear simulation, which is very important in control theory. The author then continues with fuzzy decision-making, fuzzy classification, and fuzzy pattern recognition. Fuzzy control systems gets a chapter of its own, Chapter 13. Since no text can be totally comprehensive, it has a chapter on miscellaneous topics and, finally, on fuzzy measures, including belief plausibility and probability.

It is a pleasure to read how the author develops the concept of classical logic as a juxtaposition between classical predicate and fuzzy logic. He carefully works his way through the fuzzy logic, using the well-known and classical example of the liter-full glass of water becoming empty to gradually introduce the concept of multiple-value logic. He gradually moves on to its development, then also has a section on approximate reasoning, which is presented as the ultimate goal for the theoretical foundation for reasoning about imprecise propositions, p. 199.

Although there is no complete agreement on the definition and composition of rules, the author manages to give a good perspective on each topic. Chapter 7 is full of examples that definitely help the reader to grasp the applicability of such concepts.

A nice feature at the end of each chapter is a summary and a set of references which provide the reader with additional sources. The problems are very practical.

In conclusion, this is a good textbook for a beginner, particularly for an engineer who wants to get a good perspective on the scope of fuzzy logic in engineering applications. This is one of the rare cases where the title certainly fits the content of the book.

MARIALUISA N. MCALLISTER
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System Reliability Theory: Models and Statistics Methods. *By Arnljot Høyland and Marvin Rausand.* John Wiley & Sons, New York, 1994. $69.95. viii+518 pp., cloth. ISBN 0-471-59397-4.

During the quality revolution of the 1980's, there was widespread implementation of statistical process monitoring and design of experiments for the purpose of improving the quality of manufactured products. These activities have had a positive effect on product reliability. Now, however, manufacturers have begun to focus more on implementing particular methods to improve product reliability. Høyland and Rausand provide a timely, comprehensive treatment of the modern theory of system reliability. The ideas and methods presented in the book will be important to practicing professionals as well as to students and others wanting an introduction to this interesting, important subject.

I enjoyed reading the book. I quickly gained an appreciation for the combination of practical and technical knowledge exhibited in the writing. New ideas are always motivated by clear real or realistic examples. The development is orderly and the writing style is clear and concise. Each chapter concludes with a number of interesting real/realistic problems that allow the reader to apply and explore the ideas that were developed in the chapter.

Most of the book deals with advanced probability models for reliability. The authors do, however, provide some coverage of analysis of reliability data. The mathematical level is higher than the standard engineering-oriented textbooks on the same subject. Readers without a strong calculus-based course in probability will have to struggle through some of the technical developments. Mathematical tools like matrix algebra, Laplace transform, and limiting arguments are used throughout. Høyland and Rausand cover all of the standard topics that one would expect to find in a book on system reliability. The book, however, is considerably broader than previous books in the area.

Chapter 1 introduces concepts and technical terms associated with reliability, quality, availability, maintainability and so on, and provides a discussion of different kinds of failures and ideas behind construction of models.

Chapter 2, entitled "Failure Models," introduces concepts of failure time models and gives details on most of the standard and many nonstandard parametric time-to-failure distributions, including the exponential, normal, Weibull, lognormal, Pareto, Birnbaum–Saunders, inverse Gaussian, and extreme value distributions.

Chapter 3, "Qualitative System Analysis," explains the concepts and ideas behind tools to describe, qualitatively, the structural relationships among components and how these can be related to overall system reliability. Topics include failure mode and effects analysis (FMEA), fault tree analysis, and reliability block diagrams.

Assuming systems of independent components with known component reliability, Chapter 4 shows how to use the models from Chapter 3 to develop expressions for overall system reliability and other important reliability metrics like mean time to failure (MTTF), mean time between failures (MTBF), mean fractional dead time (MFDT), availability, and so on. The authors include a particularly interesting discussion of possible periodicities that can arise in system maintenance/modeling and which require special consideration (some commonly used limiting results, for long system-operation times, require that periodicities disappear, but often they do not do so in practice because of seasonal differences or maintenance schedules).

Chapter 5 describes and illustrates measures of component importance within a particular system. Such measures help design engineers to decide where, in a large system, effort should be expended to help improve overall system reliability (e.g., low reliability components in critical positions tend to have the most importance). This chapter begins with "Birnbaum's measure," based on the partial differential approach of classical sensitivity analysis. Other methods that are also discussed and compared in a sequence of examples are "criticality importance," the "Vesely–Fussell measure," and "improvement-potential."

Chapter 6 introduces state-space concepts and describes and illustrates the use of Markov models for reliability. These models allow systems to have more than two states. Both time-dependent and asymptotic solutions are given. Technical methods used in this chapter are standard but will be interesting to those who have not seen Markov models used in reliability/availability applications. The authors, for example, show how to usefully define state spaces to extend the independent component models that were assumed in Chapter 4.

In Chapter 7 the authors introduce counting process models and show how they can be used to model a sequence of failure/repair cycles over time. The models are motivated and illustrated with repairable system data. All of the standard reliability/point process models are covered: the homogeneous Poisson process, the renewal processes, and the nonhomogeneous Poisson process. Inferential methods are developed and illustrated for the simpler methods that can be done effectively by hand.

In Chapter 8 the authors develop a number of other methods that are useful for the highly important area of modeling systems with two or more dependent failure modes. The authors describe three classes of dependent failures: common cause failures, cascading failures, and negative dependencies. With references to material in earlier chapters, the authors develop and extend models for describing failure mode dependencies.

Chapter 9 describes and illustrates some important ideas in the area of nonparametric and parametric models and methods for life data analysis. Given that there are numerous book-length treatments of this subject, it would have to be said that this chapter is only an introduction to the area. Emphasis is on simple nonparametric methods and parametric methods for the exponential distribution—models for which analyses can be performed with simple hand computations. The treatment is, however, sufficient to give the flavor of the methods that are commonly used to make inferences from reliability data. Lawless (1982), Nelson (1982), and Crowder, Kimber, Smith, and Sweeting (1991), for example, provide more complete coverage of this important topic.

Chapter 10 introduces some of the concepts of accelerated life testing, including step-stress and progressive stress testing. Accelerated tests are run at higher than usual levels of factors like cycling rate, temperature, voltage, or pressure. Extrapolation to use conditions is done through a physically motivated or physically reasonable statistical model. This chapter focuses primarily on possible mathematical structures for regression models for life data, without much attention to the physics-of-failure side of the subject that is essential for work in applied extrapolative problems. Nelson (1990) remains the seminal reference here.

Chapter 11 introduces some of the important ideas of Bayesian analysis where prior "state of knowledge" information can be combined with new data to obtain an updated posterior distribution. The focus here is again on methods that have convenient analytical solutions. The important ideas, however, come across nicely. As computational capabilities have improved there has been a resurgence in the development and application of Bayesian methods in various areas of statistics. Among many practitioners, however, there remains healthy skepticism about the way that Bayesian methods should be used in practice. The danger, as outlined in Evans (1989), is that instead of valid prior information, the input prior to distribution will reflect an engineer's wishful thinking about reliability, leading to unreasonably optimistic reliability projections. As in accelerated testing, use of prior information can be a form of extrapolation (from past product to future product). We need to give our clients the same warning that we present in Statistics 101: extrapolation is dangerous. It should be avoided whenever possible. There are, however, applications where it is important to use available, valid, prior information, but ramifications of its inclusion should be understood by those who interpret and use the results.

This book would be an excellent text for a senior-level or a graduate-level course in which the focus is on the theory of system reliability, especially for students with strong quantitative backgrounds.

Because of its modern, comprehensive treatment of the subject, those working in the area of design engineering or reliability engineering will want to have this book on their shelves as a ready reference. In this regard, Høyland and Rausand nicely complement the recently published Pecht (1995) handbook that provides more on the qualitative and physical-engineering side of reliability.

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Numerical Solution of SDE through Computer Experiments. *By P. E. Kloeden, E. Platen, and H. Schurz.* Springer-Verlag, Berlin, 1994. $49.00. 292 pp., softcover. ISBN 3-540-57074-8.

While the numerical solution of stochastic differential equations (SDEs) driven by Brownian motion and Lebesgue measure has been long studied by engineers, its mathematical development has been slow and much fundamental work is fairly recent. Indeed, the only existing books written on the subject by mathematicians are the book of P. E. Kloeden and Eckhard Platen, *Numerical Solution of Stochastic Differential Equations* (Springer-Verlag, 1992) (see its review in *SIAM Review* 37, June 95) and the very recent book by Milstein. The book under review is intended as a companion volume to the book of Kloeden and Platen, a sort of "workbook." Nevertheless, the authors claim in the preface that it can be used independently, and I will review it with that assumption made.

The first observation is that this is not a mathematics book. Theorems are not stated and no hint of real proofs is ever given. Instead, vague statements that are often annoying replace them. For example, when presenting the Euler scheme for standard stochastic differential equations, the Euler scheme is first described, then an exercise is given, and then a comment is made: "Usually the Euler scheme gives good numerical results when the drift and diffusion coefficients are nearly constant. In general, however, it is not particularly satisfactory and the use of higher order schemes is recommended" (p. 142).

In the preceding sentences it is not clear what "good," "nearly," and "satisfactory" mean. Since earlier (p. 140) it is remarked that the Euler scheme "usually attains the order of strong convergence $\gamma = 0.5$" (which means a rate of convergence of $\frac{1}{\sqrt{n}}$ if the partition step is $\frac{1}{n}$), one could assume that either $\frac{1}{\sqrt{n}}$ is not "good," or $\frac{1}{\sqrt{n}}$ is "good," but that "usually" means that the drift and diffusion coefficients are "nearly constant." At this point the reader really has to refer to Kloeden and Platen's book, or else be content with a confusingly vague understanding. Moreover, the above example is typical.

In view of these remarks, the value of the book rests primarily with the exercises. Here an unusual approach is taken: exercises are given for the reader to use with a computer, and they consist of both a mathematically computational style and programming exercises. The programming problems are written in such a way as to be understandable to almost anyone, even those who know almost nothing about programming. There is pedagogic value in this, since it can in principle serve to dispel fear of computers by showing how simple it all is. (However, in this new age, it is not clear to me that such pedagogic tactics are still needed!) The mathematical problems are, however, mostly very easy and unimaginative. They can still be instructive, but it is a bit of a shame. There does not seem to be an explicit discussion, however, that compares the cost of a method (in terms of the expected number of calculations, for example) with its asymptotic efficiency, although of course this theme is implicitly present in the discussions. This is particularly unfortunate in Chapter 4. Here, approximation schemes are given with relatively fast orders of convergence. To use them, however, one has to simulate approximate quantities. Here one finds sentences such as "The multiple stochastic integrals here can also be approximated as in (2.3.31)–(2.3.36)" (p. 154). This is misleading, since the approximations referred to on p. 83 are complicated and tedious, and therefore the higher order and faster schemes, when combined with the necessary simulations, are in fact slower in practice.

Perhaps the most important chapter is Chapter 5, where the authors consider the problem of approximating $E\{g(X_T)\}$, where X_T is the solution of an SDE at a fixed time T. If the law of X_T were known, a Monte Carlo method would suffice. Since the law of X_T is in general unknown, one combines an approximation scheme for X_T with a Monte Carlo method. Hence the Romberg-Richardson extrapolation technique (due in the stochastic case to Talay and Tubaro) is presented, but even though it is one of the most important results in the field, no special attention is called to it. Indeed, throughout the book, if the reader were to try to find *the* best way to do something, he (she) might be frustrated: while there are often exercises of the type "Repeat Problem 4.4.2 with the above mentioned schemes" (p. 173), there is no detailed discussion about when one scheme is better to use than the other. One is left with the (false) impression that in practice one just tries a lot of them to see which is best.

In summary, then, this book does not stand up well on its own, and should be treated as a companion book to the book by Kloeden and Platen. Unfortunately, this book also suffers from some (but certainly not all!) of the problems indicated in the preceding. There is also a diskette included with the book.

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Morphological Image Operators. *By Henk J. A. M. Heijmans.* Academic Press, Boston, 1994. $95.00. 509 pp., cloth. ISBN 0-12-014599-5.

The collection of reference works in the field of mathematical morphology has been enriched with a new text, namely, *Morphological Image Operators*, by H. J. A. M. Heijmans. As expressed by its author, "its main goal is to present a rigorous mathematical treatment of morphology." This is a modest statement in view of the content of the book, which represents much more.

First, it organizes the morphological concepts according to the author's personal view, which has the great merit of clarity. The chapters follow one another in a very pedagogical manner. The risk taken in writing such a book is indeed too much rigor. Since the book deals with geometry, it is appropriate to alternate precise proofs and intuitive comments or counterexamples. The author excels in this endeavor, and he demonstrates as well his ability to go from one level of generality to another. For example, after stating a result in the complete lattice framework, he often applies it to the sets of the Euclidean or digital planes.

But reducing the book to the quality of its presentation would limit its scope. As a matter of fact, there is not a single chapter in which the author does not make his personal contribution in terms of original results. In certain chapters, such as "Adjunctions, Dilations, and Erosions" (Chap. 5), or "Lattice Representation of Functions" (Chap. 10), the author's personal contribution is preponderant.

The bibliography, concluding each chapter, is therefore presented in a more lively and critical way, which clearly outlines the evolution of ideas. I would like to make a remark, which should not be considered a critical comment, that the author has not approached all the topics connected to morphological operators. For instance, he has left aside the probabilistic aspect of the method, though it starts from morphological operators (erosion), to create new ones, derived from random models. The domain of morphological segmentation (hierarchical operators, watersheds, their links with the homotopy of functions and with connected filters, etc.) is only briefly mentioned in spite of its current growth. Finally, the algorithmic implementation of the operators described is not analyzed. In all three cases, this is a deliberate choice of the author made for pedagogical reasons, which allows him to precisely target his subject, and this is a good thing. Perhaps it would have been useful to make this choice explicit in the preface.

At whom is such a book aimed? In his preface, Heijmans does not answer the question directly, but mentions that the reader's mathematical background need not be considerable. This is perfectly true. Mathematical morphology only exploits a small part of lattice theory: in the classical work done by Birkhoff on this subject, complete lattices represent only 5% of the whole text! Therefore, the reminder of the necessary notions proposed by Heijmans in his second chapter is highly sufficient. Conversely, if one were to write a monograph on complete lattices, the major part of its content would come from mathematical morphology. It is probably one of the great qualities of the book *Morphological Image Operators* to have demonstrated this so powerfully.

The chapter titles are the following: *First Principles, Complete Lattices, Operators on Complete Lattices, Operators which are Translation Invariant, Adjunctions, Dilations and Erosions, Openings and Closings, Hit-or-Miss Topology and Semi-Continuity, Discretization, Convexity, Distance, and Connectivity, Lattice Representations of Functions, Morphology for Grey-Scale Images, Morphological Filters, Filtering and Iteration.*

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Variational Methods in Image Segmentation. *By Jean-Michel Morel and Sergio Solimini.* Birkhäuser, Boston, MA, 1995. $49.00. xvi+245 pp., ISBN 0-8176-3720-6.

Computer Vision has emerged as a very interesting area of applied mathematics. One of the most difficult and mathematically interesting problems in the subject is that of locating object boundaries in the image of a scene. This of course raises the question of defining what an object is. During the early stages of image processing, an object is taken to mean simply a subset of the image domain across which some feature of the image has fairly uniform intensity and the presence of boundaries is indicated by sharp changes in the feature intensity. The task is rendered difficult because of the presence of noise which has to be filtered out without blurring the object boundaries. This amounts to finding the best piecewise smooth approximation of a given image. In one of the simplest formulations, one looks for a piecewise constant approximation of the image. That is, find the minimizers of the energy functional

$$(1) \qquad E_0(K) = \int_{\Omega\setminus K} (u_o - g)^2\,dx + \lambda^2|K|,$$

where

Ω is the image domain, an open subset in $\mathbf{R}^2$;

g is the feature intensity, $g : \Omega \to \mathbf{R}$;

K is the union of segment boundaries, thus K is the segmenting curve;

$|K|$ is the length of the segmenting curve;

u_o is a locally constant function on $\Omega\setminus K$ whose value in each connected component of $\Omega\setminus K$ equals the average of g in that component;

λ is a weight.

Often, functional (1) is too restrictive in many applications. A more general energy functional is

$$(2) \qquad \begin{aligned} E(u, K) = {}& \sigma^2 \int_{\Omega\setminus K} |\nabla u|^2\,dx \\ &+ \int_{\Omega\setminus K} (u - g)^2\,dx + \lambda^2|K|. \end{aligned}$$

The first thing to note is that minimizing $E(u, K)$ with respect to u with K fixed is the classical elliptic boundary value problem with a unique minimizer u_K. Hence, the problem reduces to the problem of minimizing $E(u_K, K)$ with respect to K. From a practical point of view, one would like the minimizing set K to be a finite set of C^1 curves. The problem has proved difficult because of the interaction of the two-dimensional term depending on u and the one-dimensional term $|K|$.

In Part I of the book, the authors survey seemingly different approaches that have been proposed for solving the segmentation problem and argue that all of these approaches may be interpreted as variations of the basic formulation (2).

Part I includes a constructive proof of the existence of regular minimizers of the relatively simple functional (1), as a forerunner of the techniques used to analyze the more complicated functional (2).

In order to analyze the more general functional (2), one needs to define a sufficiently broad class of closed subsets of Ω so that minimizers of $E(u_K, K)$ exist, at least in some weak sense, and to then analyze to what extent such minimizers are smooth. The appropriate framework within which to study subsets of Ω is geometric measure theory. In Part II of the book, the authors provide a concrete introduction to this theory, starting with the definition of the Hausdorff measure. They then proceed to define and characterize regular and irregular sets. The important properties, namely approximate tangent spaces and projections, are proved. This is followed by the semicontinuity theorems for the Hausdorff measure. The last two chapters of Part II introduce rectifiability and prove equivalence between rectifiability and regularity.

In Part III, the theory in Part II is applied to the functional at hand. It is shown that the minimizers K of $E(u_K, K)$ satisfy certain uniform density bounds and thus are Ahlfors sets. The main tool is a series of excision lemmas which assert that if a portion of K is too sparse (density too low and thus irregular), then it may be excised. Existence of a minimizer in the class of Ahlfors sets follows. The strongest regularity result due to David and Semmes proved in the last chapter asserts that there is a uniform constant $C(\Omega)$ such that the whole segmentation K is contained in a single rectifiable curve of length $\leq C|K|$ and which is Ahlfors regular. The question whether the minimizing set K is the union of finitely many C^1 curves is still open.

The book is well written and very user-friendly. Each chapter is introduced by a brief summary of its aims. Similarly, the lemmas, propositions, and theorems are well motivated. Technically difficult proofs are illustrated by a liberal use of diagrams and augmented by intuitive arguments. The bibliography is extensive.

In summary, the book is a very readable introduction to the mathematical theory of image segmentation.

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Introduction to the Modern Theory of Dynamical Systems. *By Anatole Katok and Boris Hasselblatt.* Cambridge University Press, Cambridge, UK, \$79.95. xviii+802 pp., cloth. ISBN 0-521-34187-6. Encyclopedia of Mathematics and Its Applications, Vol. 54, 1995.

In the film *Jurassic Park*, the elderly tycoon (who builds the ill-fated dinosaur theme park somewhere off Costa Rica) introduces the Jeff Goldblum character as the resident mathematician. "I'm not a mathematician; I'm a chaotician!", objects Goldblum. My fantasy is that every self-proclaimed "chaotician" would be first a mathematician, and an excellent way to do so would be to read *Introduction to the Modern Theory of Dynamical Systems*, by Anatole Katok and Boris Hasselblatt, hereafter denoted IMTDS.

The modern theory of dynamical systems has come of age. One trivial evidence of this is that there are fewer and fewer painful references to "dynamic systems." In popular science writing, if not pop films, one even sees dynamical systems mentioned as having something to do with "chaos theory," which is a mixed blessing. What is more important, a spate of new books has appeared over the past several years which purport to introduce dynamical systems to a wider, mathematically literate audience. The subject is sufficiently vast that each book (except, perhaps, for IMTDS) develops selected topics at the expense of others.

Dynamical systems theory has no neat beginning in time, although Poincaré is often credited as the father of the subject. Colleagues in other fields sometimes ask me, in apparent annoyance, "but what exactly *is* a dynamical system? You people never define one!" One can certainly formally speak of a continuous semigroup action on a topological space, but that approach leads to the kind of abstract sterility that was the fate of a good deal of topological dynamics. I much prefer the informal, three-paragraph description in the opening paragraphs of IMTDS.

The sheer size of the subject owes, in large part, to its eclectic origins. The authors of IMTDS enumerate the following principal branches of modern dynamical systems: ergodic theory, topological dynamics, differentiable dynamics, and Hamiltonian systems. This is the only text that I know of which treats all four of these topics in considerable detail at the graduate level. To make a comparison, another graduate text which I also like, *Dynamical Systems*, by Clark Robinson (CRC Press, 1995), does not treat ergodic theory, although it does a credible job in the other areas. This may simply reflect a difference in training between graduate students in the US and, say, those of Eastern Europe. To accommodate all those topics, IMTDS is about 800 pages in length. Different instructors might well teach entirely different courses based on this book.

What some mathematicians may find wanting in IMTDS is modern applied dynamical systems, but that is not the objective of the book. A reader who knows some dynamical systems theory and wants to learn about recent applications would be better served by reading SIAM journals or the excellent little book *Nonlinear Dynamics and Chaos*, by Steven Strogatz (Addison-Wesley, 1994). But "applied" depends on one's definitions. There is quite a bit of material on billiard problems, the geodesic flow, and Hamiltonian systems in general, which I would call classical mathematical physics or mechanics. If an applied mathematician wanted to use dynamical systems to attack a physical problem, the ideal preparation might be to read through this book.

The general level of writing for this text is about right. The introductions to chapters and sections are quite readable and what proofs I read seemed clear and precise without being tedious. Some topics would be too sophisticated for many graduate students, but here the instructor can pick and choose something more appropriate. One novelty is the emphasis on "cocycles." Cocycles come up whenever one needs to solve an equation of the form $g(x) = R\phi(F(x)) - \phi(x)$, where R is linear and F is a discrete map. Such "cohomological equations" arise in reparameterizing ODEs, in structural stability problems, and in the construction of minimal dynamical systems on the torus.

The notes section at the end of the book is complete and quite helpful. There are hints and answers provided for a good percentage of the problems in the book. The problems range from fairly straightforward ones to results that I remember reading in research papers over the last 10–20 years. The appendix includes readable introductions to those portions of topology, functional analysis, surface theory, measure theory, differential geometry, homology, and Lie theory that play a role in dynamical systems. I recommend the text as an exceptional reference which is still a bargain at \$80.

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Approximation Procedures in Nonlinear Oscillation Theory. *By Nikolai A. Bobylev, Yuri M. Burman, and Sergey K. Korovin.* Walter de Gruyter, Berlin, 1994. DM 158. xi+272 pp., cloth. ISBN 3-11-014132-9.

This is an excellent book on nonlinear analysis, approximate methods, and applications to oscillation theory. The book is written on the highest mathematical level, but also in such a way that engineers can enjoy it. A mathematician will find vivid models of oscillation theory; an engineer will find ways to apply deep mathematical analysis to engineering problems; both will see how approximate methods can be developed to meet the needs of concrete problems.

The book consists of three chapters. In Chapter 1 ("Basic Concepts"), oscillatory models and a description of approximate methods to find the oscillations from these models are presented. The main models can be described by the following systems of differential equations:

$$\frac{dx}{dt} = f(t, x), \tag{1}$$

$$\frac{dx}{dt} = f(x), \tag{2}$$

$$L\left(\frac{d}{dt}\right)x(t) = M\left(\frac{d}{dt}\right)(f(t, x(t)) + u(t)), \tag{3}$$

where $f(t, x)$ and the input $u(t)$ are T-periodic in t and $L(p)$ and $M(p)$ are polynomials of degree l and m, respectively, $l > m$. A T-periodic output $x(t) : [0, T] \to \mathbb{R}^N$ is looked for. In the case of the autonomous system (2), the period T is also to be determined. Integral and integrofunctional reformulations of problems (1)–(3) are given. The following methods to find $x(t)$ from (1)–(3) or the corresponding integral equations are described: harmonic balance (= Galerkin method with trigonometric basic functions), quadratures, collocation, finite differences.

Chapter 2 ("Existence Theorems for Oscillatory Regimes") contains a full exposition of the theory concerning the degree of a mapping, rotation of a completely continuous vector field, fixed point principles (Brouwer, Browder, Schauder, and Leray-Schauder among others), with applications to oscillatory systems.

Chapter 3 ("Convergence of Numerical Procedures") is the longest in the book. The authors examine the projection and factor methods for general nonlinear problems and apply the results to the oscillatory problems and approximate methods described in Section 1. The global convergence analysis is presented on the basis of the rotation notion (or the notion of the degree of a mapping). In the case of "nondegenerate" isolated solution, a priori and a posteriori error estimates are derived.

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SELECTED COLLECTIONS

Approximation and Computation. *Edited by R. V. M. Zahar.* Birkhäuser, Boston, MA, 1994. $98.00. xlvi + 591 pp., hardcover. ISBN 0-8176-3753-2.

During December of 1993 a conference in honor of Walter Gautschi was held at Purdue University. This Festschrift is the proceedings of that conference. The conference was organized around four main themes to which Professor Gautschi has made many important and lasting contributions: approximation theory, orthogonal polynomials, quadrature, and special functions. Thirty-eight refereed papers covering all aspects of the themes above, as well as some interdisciplinary papers, fill out the volume.

Approximation Theory, Wavelets and Applications. *Edited by S. P. Singh, A. Carbone, and B. Watson.* Kluwer, Dordrecht, the Netherlands, 1995. $254.00. xxiii + 573 pp., hardcover. ISBN 0-7923-3334-9.

This proceedings of a NATO Advanced Study Institute, held at Maratea, Italy, during May 1994 is dedicated to Professor E. W. Cheney. The 40 papers cover the main topics of multivariate approximation, the theory of splines, spline wavelets, polynomial and trigonometric wavelets, interpolation theory, polynomial and rational approximation, and applications.

Asymptotic Methods for Elastic Structures. *Edited by P. Ciarlet, L. Trabucho, and J. Viaño.* Walter de Gruyter, Berlin, 1995. $128.95. 291 pp., hardcover. ISBN 3-11-014731-9.

The 21 contributions in this volume give an overall picture of current developments in asymptotics for elastic structures and their relationship with other areas of applied mathematics, such as numerical analysis, controllability, homogenization, and optimization. The papers comprise the proceedings of an international conference held at Lisbon, Portugal during October 1993.

Asymptotic Theories for Plates and Shells. *Edited by R. P. Gilbert and K. Hackl.* Longman, Harlow, England, 1995. £23.00. i + 131 pp., softcover. ISBN 0-582-24875-2.

The papers in this volume were presented at a minisymposium associated with SIAM's 40th Anniversary Meeting held at Los Angeles during June 1992. The theme of the minisymposium was the theory of plates and shells, with special emphasis on the treatment of different materials and the nonlinearities involved. Of particular concern are rigorous derivation of plate and shell models and analytic treatments using asymptotic methods, formal expansions, homogenization, and two-scale convergence.

Computation and Control IV. *Edited by K. Bowers and J. Lund.* Birkhäuser, Boston, MA, 1995. $98.00. xi + 356 pp., hardcover. ISBN 0-8176-3774-5.

This proceedings of the Fourth Bozeman Conference on Computation and Control, held at Montana State University during August 1994, contains 23 papers. The general theme of the papers is the problem of developing rigorous numerical methods and computational tools for control design and analysis.

Electric and Magnetic Fields: From Numerical Models to Industrial Applications. *Edited by A. Nicolet and R. Belmans.* Plenum, New

York, 1995. \$105.00. xii + 376 pp., hardcover. ISBN 0-306-44991-9.

Eighty-five papers from the Second International Workshop on Electric and Magnetic Fields held at the Catholic University of Leuven, Belgium during May 1994 are contained in this volume. The main topics of the collection are coupled problems, CAD and CAM applications, three-dimensional eddy current and high frequency problems, optimization, and application-oriented numerical problems.

Enabling Technologies for Petaflops Computing. *Edited by T. Sterling, P. Messina, and P. Smith.* MIT Press, Cambridge, MA, 1995. \$26.95. x + 180 pp., softcover. ISBN 0-262-69176-0.

A Petaflops computer (10^{15} floating point operations per second) is for now just a gleam in the computer scientist's eye. Making such a computer work will require architectures and software that are radically different from anything that is envisioned today. The papers in this volume report on key findings of a workshop held at Pasadena in February 1994 which assessed the technologies needed for a Petaflops computer and set near-term research directions.

Free Boundary Problems: Theory and Applications. *Edited by J. Diaz, M. Herrero, A. Liñan, and J. Vazquez.* Longman, Harlow, England, 1995. £23. iii + 219 pp., softcover. ISBN 0-582-25645-3.

The papers in this proceedings of a conference held at Toledo, Spain in June 1993 are grouped under the headings free boundary problems in European industry, mean curvature flows, phase transitions and materials sciences, fluid mechanics, combustion problems, free boundary problems in U.S. industry, mathematical developments in free boundary problems, free boundaries in evolution problems, and control and identification.

The Interplay between Differential Geometry and Differential Equations. *Edited by V. V. Lychagin.* American Mathematical Society, Providence, RI, 1995. \$98.00. ix + 294 pp., hardcover. American Mathematical Society Translations, Series 2, Vol. 167. ISBN 0-8218-0428-6.

The purpose of this book is to emphasize the advantage of the algebraic geometry approach to nonlinear differential equations, including applications of symplectic methods and the discussion of quantization problems. One of the common features of the majority of the papers is the systematic use of the geometry of jet spaces.

Inverse Problems and Applications to Geophysics, Industry, Medicine and Technology. *Edited by D. D. Ang, R. Gorenflo, R. Rutman, T. Van, and M. Yamamoto.* Publications of the HoChiMinh City Mathematical Society, Vol. 2. HoChiMinh City, Viet Nam, 1995. x + 226 pp., softcover. No price or ISBN number given.

This proceedings of the International Workshop on Inverse Problems, held at HoChiMinh City in January 1995, contains the full texts of 22 papers and several abstracts of papers on all aspects of the theory and applications of inverse problems. The emphasis is on approximation theory, computational algorithms, and applications.

Inverse Problems in Diffusion Processes. *Edited by H. W. Engl and W. Rundell.* Society for Industrial and Applied Mathematics, Philadelphia, PA, 1995. \$58.00. xi + 232 pp., softcover. ISBN 0-89871-351-X.

This collection of eleven expository papers encompasses both the theoretical and applied sides of inverse problems in diffusion processes. The emphasis is on applications in heat transfer, flow in porous media, nondestructive evaluation and other diffusion processes where parametric estimation is a critical factor. The papers are based on lectures presented at a GAMM/SIAM-sponsored conference held at Lake St. Wolfgang, near Strobl, Austria, in June 1994.

Mathematical and Numerical Aspects of Wave Propagation. *Edited by G. Cohen.* Society for Industrial and Applied Mathematics, Philadelphia, PA, 1995. \$106.00. xii + 808 pp., softcover. ISBN 0-89871-350-1.

This volume comprises the proceedings of the Third International Conference on Mathematical and Numerical Aspects of Wave Propagation which was held at Mandelieu-La Napoule, France during April 1995. Topics covered include boundary integral equations, numerical methods, absorbing boundary conditions, guided waves, asymptotic methods, inverse problems and optimal control, domain decomposition methods, nonlinear waves, electromagnetism, water waves, and parallel processing.

The Mathematics of Generalization. *Edited by D. Wolpert.* Addison-Wesley, Reading, MA, 1995. $31.95. xvii + 441 pp., softcover. ISBN 0-201-40983-6.

The papers in this proceedings of the SFI/CNLS Workshop on Formal Approaches to Supervised Learning held at the Santa Fe Institute in the summer of 1992 have the goal of opening lines of communication among the various fields concerned with supervised learning, such as neural nets, conventional Bayesian statistics, conventional sampling theory statistics, computational learning theory, artificial intelligence, and machine learning.

Parallel Processing of Discrete Optimization Problems. *Edited by P. Pardalos, M. Resende, and K. Ramakrishnan.* American Mathematical Society, Providence, RI, 1995. $89.00. xiv + 374 pp., hardcover. DIMACS Series in Discrete Mathematics and Computer Science, Vol. 22. ISBN 0-8218-0240-2.

This volume is the proceedings of a DIMACS workshop held at Rutgers University in April 1994. The papers cover a wide spectrum of the most recent algorithms and applications in parallel processing of discrete optimization and related problems. Topics include parallel branch and bound algorithms, scalability, load balancing, parallelism and irregular data structures, and scheduling task graphs on parallel machines.

Recent Developments in Evolution Equations. *Edited by A. McBride and G. Roach.* Longman, Harlow, England, 1995. £23. iv + 251 pp., softcover. ISBN 0-582-24669-5.

The 28 papers in this volume comprise the proceedings of a conference held at the University of Strathclyde in July 1994. The main topics are semigroups and related subjects connected with applications to partial differential equations of evolution type.

Symposia Gaussiana, Symposium A: Mathematics and Theoretical Physics. *Edited by M. Behara, R. Fritsch, and R. Lintz.* Walter de Gruyter, Berlin, 1995. DM328. xx + 745 pp., hardcover. ISBN 3-110-14476-X.

The 54 papers in this volume are grouped under the headings mathematical education, history of mathematics, mathematical logic, algebra and number theory, geometry, analysis of several complex variables, algebraic topology, quantum groups and q-deformation, computational physics, relativistic celestial mechanics, and Gauss and geomagnetism.

Symposia Gaussiana, Symposia B: Statistical Sciences. *Edited by V. Mammitzsch and H. Schneeweiss.* Walter de Gruyter, Berlin, 1995. DM268. x + 342 pp., hardcover. ISBN 3-110-14412-3.

This volume contains a selection of refereed papers presented at the Second Gauss Symposium grouped under the headings probability theory, probabilistic expert systems, statistical decision theory, simulation and resampling, linear models and design of experiments, and general methods and applications.

Variational Inequalities and Network Equilibrium Problems. *Edited by F. Giannessi and A. Maugeri.* Plenum, New York, 1995. $89.50. xii + 305 pp., hardcover. ISBN 0-306-45007-0.

The 22 papers in this volume explore the theory of variational and quasi-variational inequalities and applications of the theory to equilibrium problems in transportation and computer and electrical networks to market behavior and to bi-level programming. The papers were presented at a conference held at Erice, Italy during June 1994.

ERRATA

In *SIAM Review*, Vol. 36, No. 1, p. 117, Walter de Gruyter is listed as co-author with Klaus Deimling of the book *Multivalued Differential Equations* (de Gruyter, Berlin, 1992). Professor Deimling is the sole author of the book and Walter de Gruyter is the book's *publisher*.