# Writing With Mathematics: Format, and Mood

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## 1 Formatting

The shorthand nature of mathematical notation allows formulation of extremely complicated statements. Various conventions for formatting have come about to help readers digest such statements.

### 1.1 Center for focus

Although equations can be clauses in larger sentences, for the sake of readability and emphasis, one ought to put important equations on separate lines, centered. The following is an example of such writing.

> The total revenue, $R$, made from selling widgets is given by the equation
> $$R = pq,$$
> where $p$ is the price at which each widget is sold and $q$ is the number of widgets sold. Based on past experience, we know that when widgets are priced at 15 each, 2000 widgets will be sold. We also know that for every dollar increase in price, 150 fewer widgets are sold. Hence, if the price is increased by $x$ dollars, then the revenue is
> $$\begin{aligned} R &= (15 + x)(2000 - 150x) \\ &= -150x^2 - 250x + 30,000. \end{aligned}$$

### 1.2 Carriage return for readability

Not doing so forces the readers eyes to move around the page in unfamiliar ways. The following is a slight offense.

> $$\text{The system of equations} \quad \begin{array}{c} x + y = 1 \\ x - y = 1 \end{array} \quad \begin{array}{c} \text{is equivalent to} \\ \text{the system} \\ \text{of equations} \end{array} \quad \begin{array}{c} x + y = 1 \\ 2x = 2 \end{array} \; .$$

When one sees this kind of writing one sometimes worries that one is not reading things in the right order. Clear things up by respecting the carriage return conventions.

The system of equations

$$x + y = 1$$

$$x - y = 1$$

is equivalent to the system

$$x + y = 1$$

$$2x = 2.$$

## 1.3 Placement of symbols in carriage returns

Note that, conventionally, equal signs, implies signs, etc go on the next line. The following is a violation of that convention.

$$\begin{aligned} R = \quad & (15 + x)(2000 - 150x) = \\ & -150x^2 - 250x + 30,000. \end{aligned}$$

The following is in agreement with that convention.

$$\begin{aligned} & 2y = 12x + 24 \\ \Longrightarrow \quad & y = 6x + 12. \end{aligned}$$

If a single algebraic expression is too long for a line, the operation connecting two lines goes on the latter line.

$$\begin{aligned} (A + B)^3 = \quad & A^3 + AAB + ABA + BAA \\ & +ABB + BAB + BBA + BBB. \end{aligned}$$

# 2 Mood

Three moods dominate the english language.

The **imperative mood** if for issuing commands.

> First write $2^2$ and then rewrite this as $2 \cdot 2$. Last, multiply 2 and 2.

This mood is appropriate for communicating algorithms (sequences of steps used to cary out tasks.) But be aware that describing an algorithm and describing why an algorithm performs its intended task are very different things. For a math class, you need to describe why algorithms work; you need to state facts not instructions.

The **subjunctive mood** is for discussing hypothetical situations.

> If $f(x)$ were $x^2$ then $f(2)$ would be 4.

Consideration of hypotheticals is high level of thinking that requires considerable effort; there is no need to force your reader into this mode of thinking to communicating facts about mathematics. The phrases "were" and "would be" in above example should both be replaced with "is" to put this statement in the declarative mood.

The **declarative mood** is for stating facts.

> If $f(x) = x^2$ for all numbers $x$ then $f(2) = 2^2 = 4$.

Use the declarative mood when writing *about* mathematics; the logic side of mathematics deals with relationships between facts.

## 2.1 Claim

Clarify to your reader the reason for their reading before they read it by informing them that you are making a claim. A claim is a statement; it is either true or false. You reader will read on to see how you argue that your claim is a true statement.

I claim that for any natural number $k$, the sum of the first $k$ odd natural numbers is $k^2$. In particular, the sum of the first 2 natural numbers is
$1 + 3 = 4 = 2^2$
and the sum of the fist 3 odd natural numbers is
$1 + 3 + 5 = 9 = 3^2$.
In general, if the statement is true for some particular integer $k$ so that
$\sum_{i=1}^{k}(2i - 1) = k^2$
then
$\sum_{i=1}^{k+1}(2i - 1) = \sum_{i=1}^{k}(2i - 1) + [2(k + 1) - 1] = k^2 + 2k + 1 = (k + 1)^2$
so the statement is true for the particular natural number $k+1$ also. Then, by the principle of induction, for any natural number $k$, the sum of the first $k$ odd natural numbers is $k^2$.

## 2.2 Algorithms vs Flows of Logic: Process vs fact

The following is a common form of response from a novice to writing with math.

Why is it that the sum of two odd numbers is even?

First, pick two odd numbers $m$ and $n$.
Next, find two numbers $a$ and $b$ such that $m = 2a+1$ and $n = 2b+1$.
Next, add $m$ and $n$.
Next, write the result $2a + 2b + 2 = 2(a + b + 1)$.

This string of imperatives reads like a set of instructions given to a computer. If your audience is human, a flow of logic is more appropriate.

Why is it that the sum of two odd numbers is even?

If $m$ and $n$ are odd numbers then there exist integers $a$ and $b$ such that $m = 2a + 1$ and $n = 2b + 1$. Therefore

$$m + n = 2a + 2b + 2 = 2(a + b + 1)$$

is a multiple of 2, and thus even.

# 3 Passive Voice

Last century, there was a movement toward eliminating the active voice from scientific writing. There has been pushback as the resultant writing was dry, boring, pompous, and just plain bad. I do not care to participate in the ongoing debate.

# 4 Less than sophisticated addictive phrases

Here, I have collected some phrases that I frequently see in my students writing that strike me as unsophisticated. I do this so that you can look at your own writing and think over which phrases you might habitually use that are best left in elementary school before they end up on a job application.

-"We are given ..."
"plug it in"
"Solve it out"
-"I would"
"The answer is..."
-"3+4 looks like 7."
False urgency, or no other way mentality "We must"
Indicating canceling with slashes.
Chimeras "The value of f at 3 = 5."
Future subjunctive? "1+x+1 would be x+2."
A solution is a time? "A solution to an equation is when you substitute..."

# 5 Literacy

A college degree should indicate that the degree holder has developed and demonstrated the ability to communicate about technical, complicated, nuanced topics and also to develop and communicate the holder's own ideas. This kind of communication requires one to think hard about one's audience. The difficulty in verbalizing thoughts is not in putting your ideas into words such that you can read what you wrote, but in putting your ideas into words that other people can not misunderstand.

Literacy is not just about the ability to read. It is the ability to read and write. It is my opinion that college courses should require students to read, comprehend, write, and be comprehended. This is an important set of skills to have in California, 2014; our economy and job markets center more and more on technical skills. To participate, you must be able to read and write about technical material. Development of these skills takes tremendous amounts of practice, but I hope these handouts and homeworks have helped to provide you with a sense of direction in this process.

-Professor Cherney,