LECTURE #1

Linear Algebra

MAT 22A

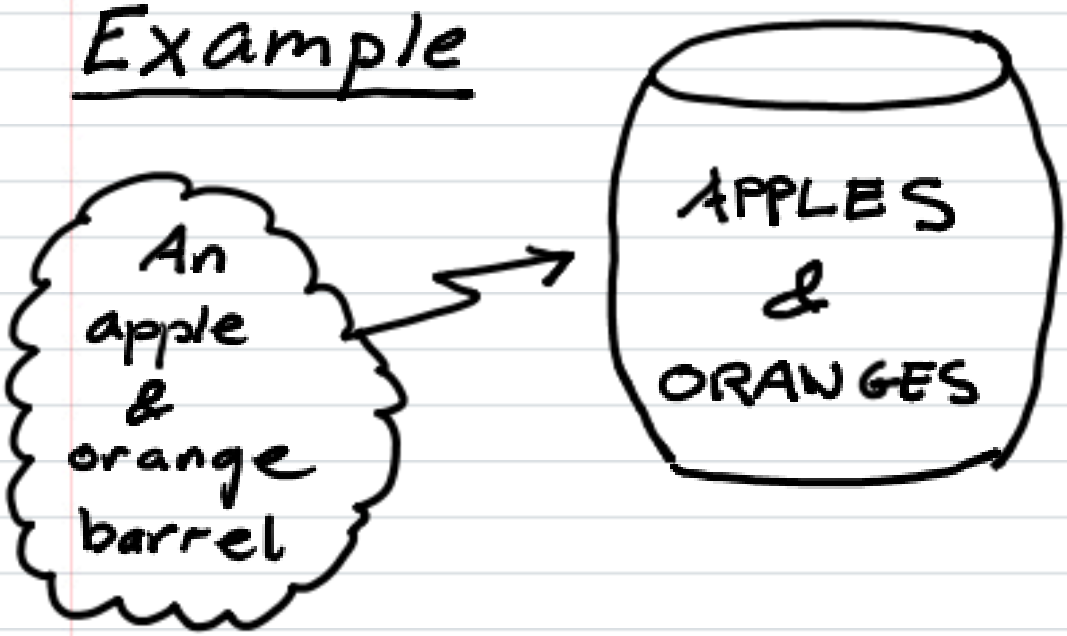
Q. WHAT IS THE COURSE ABOUT?

A. LINEAR SYSTEMS,
VECTOR SPACES &
LINEAR TRANSFORMATIONS

Q. WHAT SKILLS WILL I LEARN?

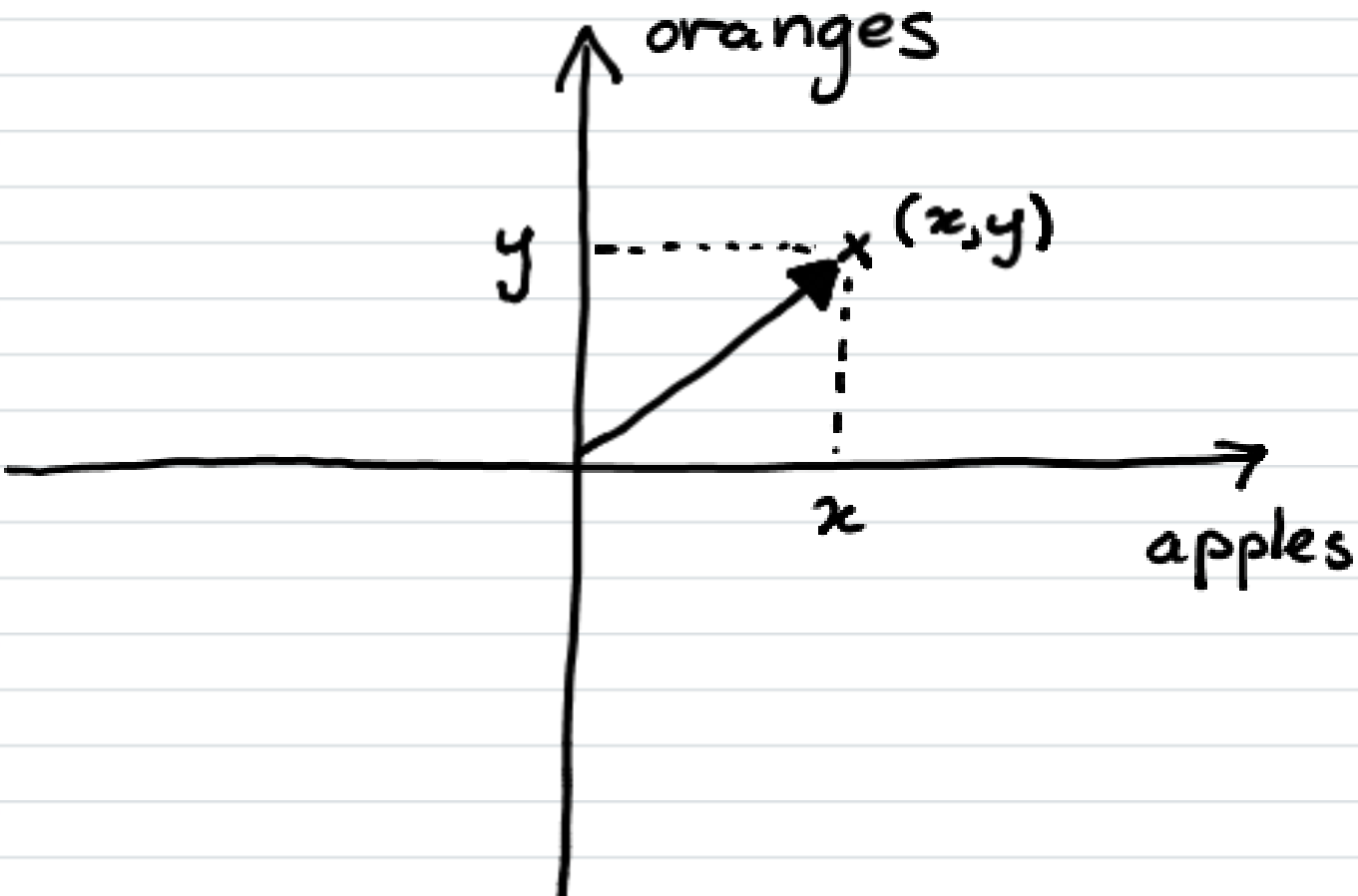
A. SOLVING LINEAR SYSTEMS
& MANIPULATING MATRICES

Example

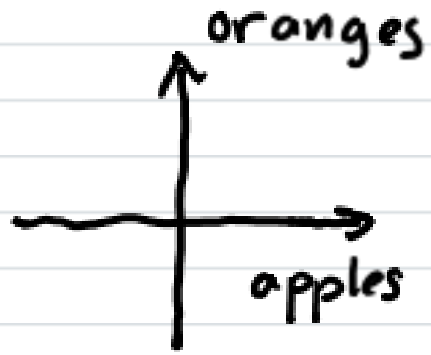


Mathematical representation

$$\begin{cases} x = \#\text{ of apples} \\ y = \#\text{ of oranges} \end{cases}$$

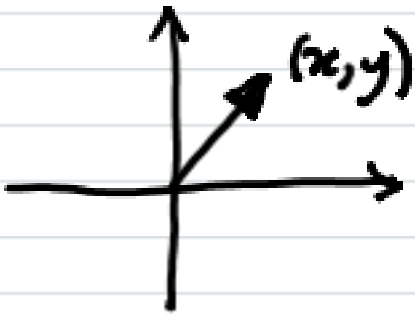


Some names:



= A Vector Space

orange-apple plane



= A Vector

the ordered pair of numbers (x, y)

A linear system

Q. There are 27 pieces of fruit and twice as many oranges as apples, what is in the barrel?

A. $$\begin{cases} x + y = 27 \\ y = 2x \end{cases}$$

$\Rightarrow x + 2x = 3x = 27 \Rightarrow x = 9$

$\Rightarrow y = 2 \cdot 9 = 18$.

9 apples & 18 oranges

The above equations are a linear system. Only multiples of the first power of the unknowns x and y appear.

Rewrite our equations tidily

$$x + y = 27$$

$$2x - y = 0$$

We want to recognize the vector (x, y)

$$\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 27 \\ 0 \end{pmatrix}$$

a MATRIX to store the coefficients

a vector

another vector

Rule for matrices multiplying vectors

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

Linear transformations:

$$\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 27 \\ 0 \end{pmatrix}$$

MATRIX VECTOR VECTOR

The matrix maps vectors $\longrightarrow$ vectors and is an example of a linear transformation.

NEXT TIME:

Gaussian Elimination

A method for solving linear systems.

Lecture 1 Review Questions

1. Let M be the matrix

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

and v, u the vectors.

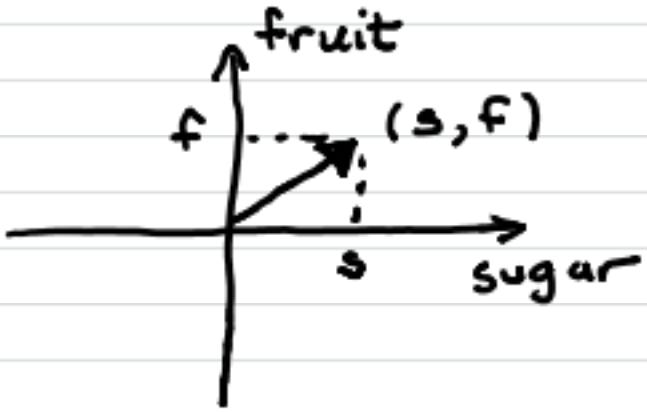
$$v = \begin{pmatrix} x \\ y \end{pmatrix} \qquad u = \begin{pmatrix} w \\ z \end{pmatrix}$$

(i) Propose a definition for $u+v$.

(ii) Check that your definition obeys

$$Mv + Mu = M(v+u)$$

2. Pablo is a biologist and he knows that oranges have twice as much sugar as apples. Therefore he represents the barrel by



$\begin{cases} s = \text{sugar} \\ f = \text{total fruit} \end{cases}$

Find a linear transformation relating Pablo's representation to the one in Lecture. Write your answer as a matrix.