A typical game of Tic-Tac-Toe is played by two people. Each take turn to mark in a three-by-three, two-dimensional grid. The objective of the game is to place three marks of the same kind in a row either in a horizontal, vertical, or diagonal direction. The following picture from Wikipedia demonstrates one of the possible sequences leading to a winning condition in the horizontal direction.

We will expand the popular definition of Tic-Tac-Toe to k-Tic-Tac-Toe, a game of two or more players taking turn to mark in a k-dimensional space, each dimension with a set of domain $\mathbb{Z}_3 = \{0, 1, 2\}$. Notice we can apply our experience with $\mathbb{Z}_2 = \{0, 1\}$. If we define vector addition and scalar multiplication carefully, which we will discuss later, we will have a vector space $\mathbb{Z}_3^k$. The number of grids in a k-dimensional game is $3^k$. A player wins a game if three marks of the same kind is connected in a line.

**0-Tic-Tac-Toe and 1-Tic-Tac-Toe**
Since $3^0 = 1$, o-Tic-Tac-Toe has to be played within one grid, which is not possible. Similarly, $3^1 = 3$, a player cannot satisfy the winning condition without violating the rules in a 1-Tic-Tac-Toe.

**2-Tic-Tac-Toe**
This is the original two-dimensional Tic-Tac-Toe. We start our analysis by introducing position vectors that describes the location of a mark, vector addition and scalar multiplication rules in this vector space, and the conditions under which these vectors must meet in order to qualify as a wining set.

The tables to the left illustrate the addition and scalar multiplication rules to maintain the vector space property. These rules are analogous to those of the bit matrices.
This general definition works for all the k-dimensional Tic-Tac-Toe’s. For example, in the two-dimensional case, the set \{(2, 0), (1, 1), (0, 2)\} wins the game because each element is in the set \{(1, 1) + k[(2, 0) - (1, 1)]\}. The choice of position vectors is arbitrary. The set \{(0, 2) + k[(1, 1)-(0, 2)]\} and the set \{(2, 0) +k[(2, 0)-(0, 2)]\} contains the solution as well.

**3-Tic-Tac-Toe**

In three-dimensional Tic-Tac-Toe, which has \(3^3 = 27\) grids, we use a third component to indicate which plane a given mark is. Using the same winning condition, which we will restate here: A set of position vectors locating the same kind of mark \(W = \{W_1, W_2, W_3\}\) wins the game if and only if \(W\) is a subset of \(S = \{I + kD\}\), where \(I\) is an element in \(W\), \(D\) is the difference vector between two distinct element in \(W\), and \(k\) is a real number.

Notice the solutions from a 2-Tic-Tac-Toe game is valid in 3-Tic-Tac-Toe space. When \(n > k\), the solution from a k-Tic-Tac-Toe game works for any n-Tic-Tac-Toe game; the elements of the ‘lower dimension’ winning set in the n-Tic-Tac-Toe game will just have the same numbers from the \(k+1^{st}\) components on.
A fourth component of the position vector is introduced to describe which cube a mark is. Though we are now entering a four-dimensional game, the same condition for winning the game still applies. Examples of winning sets include \{(0, 0, 2, 2), (1, 1, 1, 1), (2, 2, 0, 0)\}, \{(0, 2, 2, 2), (1, 1, 2, 1), (2, 0, 2, 0)\}, and \{(2, 2, 1, 0), (2, 2, 1, 1), (2, 2, 1, 2)\} as the following illustrations show respectively.

In conclusion, this condition to win continues for higher-dimensional Tic-Tac-Toe games as long as the three marks of the same kind form a valid subset in the set of winning ‘lines.’

References

Dr. George's Science Web
Tic-tac-toe @ Wikipedia