CSE 788.01 - Sparse representations and compressive sensing
Problem Set 1
Due lecture on April 17th

1. A Hadamard matrix is an n-by-n matrix with entries in $\{-1, 1\}$ so that all columns are orthogonal. Show that if there is a Hadamard matrix of size $n \geq 1$, then n must be 1, 2 or a multiple of 4.

2. Prove the following converse to the necessary condition for recovery of sparse signals that we discussed in the second lecture: Suppose that an m-by-n matrix A has the property that every $2k$ columns are linearly independent. Show that for any vector $y \in \mathbb{R}^m$ there exists at most one k-sparse signal $x \in \mathbb{R}^n$ such that $y = Ax$.

3. A linear program is an optimization problem of the form: For an m-by-n matrix A and vectors $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$,

$$\begin{aligned} &\min \sum_i c_i x_i \\ \text{subject to} \quad &Ax \leq b. \end{aligned}$$

   (where the inequality is coordinate by coordinate)

   Show that the following optimization problem (l_1 reconstruction) can be rewritten as an equivalent linear program:

$$\begin{aligned} &\min \|x\|_1 \\ \text{subject to} \quad &Ax = b. \end{aligned}$$

   Equivalence here means than from a solution of the equivalent linear program we can get a solution to the original problem efficiently. Hint: use auxiliary variables.