1. A Hadamard matrix is an $n$-by-$n$ matrix with entries in $\{-1, 1\}$ so that all columns are orthogonal. Show that if there is a Hadamard matrix of size $n \geq 1$, then $n$ must be 1, 2 or a multiple of 4.

2. Prove the following converse to the necessary condition for recovery of sparse signals that we discussed in the second lecture: Suppose that an $m$-by-$n$ matrix $A$ has the property that every $2k$ columns are linearly independent. Show that for any vector $y \in \mathbb{R}^m$ there exists at most one $k$-sparse signal $x \in \mathbb{R}^n$ such that $y = Ax$.

3. A linear program is an optimization problem of the form: For an $m$-by-$n$ matrix $A$ and vectors $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$,

$$\min \sum_i c_i x_i$$

subject to $A x \leq b$.

(where the inequality is coordinate by coordinate)

Show that the following optimization problem ($l_1$ reconstruction) can be rewritten as an equivalent linear program:

$$\min \|x\|_1$$

subject to $A x = b$.

Equivalence here means than from a solution of the equivalent linear program we can get a solution to the original problem efficiently. Hint: use auxiliary variables.