CSE 680 - Problem Set 2
Due lecture on October 22nd

Collaboration is permitted; looking for solutions from external sources (books, the web, etc.) is prohibited.

1. Show that, under the assumptions of the master method, the regularity condition in case 3 always holds whenever $f(n) = n^k$ and $f(n) = \Omega(n^{\log_b a+\epsilon})$ for constant $\epsilon > 0$. (The regularity condition is: $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all n larger than some constant $n_0 \in \mathbb{N}$.)

2. Give the asymptotic running time of the following algorithms in Θ notation. Justify your solution.

   (a)
   ```
   function func1(n)
   s = 0
   for i = 2*n to n^2 do
       for j = i to n^2 do
           s = s + j - i
       endfor
   endfor
   return(s)
   ```

(b)
```
function func2(n)
s = 0
for i = 1 to n^2 do
    j = i^3
    while j > 10 do
        j = floor(j/5)
        s = s + j - i
    endwhile
endfor
return(s)
```

3. Write a recurrence relation describing the worst case running time of each of the following algorithms and determine the asymptotic complexity of the function defined by the recurrence relation. Justify your solution. Assume that all arithmetic operations take constant time.

   (a)
   ```
   function func3(n)
   if n <= 10 then return(n)
   x = floor(n/7)
   x = x + func3(floor(3n/4))
   return(x)
   ```

   (b)
   ```
   function func4(A,n)
   /* A is an array of n integers */
   if n <= 2 then return (A[1])
   for i = 1 to floor(n/2) do
       A[i] = A[i] + A[i+1]
       B[i] = A[i] - A[i-1]
       C[i] = A[i] + A[2*i]
   endfor
   x = 0
   x = x + func4(A, floor(n/3))
   x = x + func4(B, floor(n/3))
   x = x + func4(C, floor(n/3))
   return(x)
   ```

4. V. Pan has discovered a way of multiplying 68×68 matrices using 132,464 multiplications, a way of multiplying 70×70 matrices using 143,640 multiplications, and a way of multiplying 72×72 matrices using 155,424 multiplications. Which method yields the best asymptotic

running time when used in a divide-and-conquer matrix-multiplication algorithm? How does it compare to Strassen's algorithm?