CSE 680 - Problem Set 3
Due lecture on November 19th

Collaboration is permitted; looking for solutions from external sources (books, the web, etc.) is prohibited.

1. (a) What is the running time of (deterministic) quicksort when all elements of the input are equal? Explain.
   (b) Show that the following problem can be solved in time \( O(n \log n) \):
   Given an array of \( n \) numbers, determine whether all the elements in the array are distinct.

2. (a) What is the smallest possible depth of a leaf in a decision tree for a comparison sort? Explain.
   (b) Describe an algorithm that, given \( n \) integers in the range 0 to \( k \), preprocesses its input and then answers any query about how many of the \( n \) integers fall into a range \([a..b]\) in \( O(1) \) time. Your algorithm should use \( \Theta(n+k) \) preprocessing time. (Hint: counting sort)

3. The \( k \)th quantiles of an \( n \)-element set are the \( k - 1 \) order statistics that divide the sorted set into \( k \) equal-sized sets (to within 1). For example, the 4th quantiles of an array of size 64 are the 16th, 32nd and 48th order statistics. Give an \( O(n \log k) \)-time algorithm to list the \( k \)th quantiles of a set.