CSE 680 - Problem Set 4
Due lecture on December 3rd

Problem numbers are from the second edition or the third of "Introduction to algorithms". If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, etc.) is prohibited.

1. (a) Suppose that we are storing a set of n keys into a hash table of size m. Show that if the keys are drawn from a universe U with $|U| > mn$, then U has a subset of size n consisting of keys that all hash to the same slot, so that the worst-case searching time for hashing with chaining is $\Omega(n)$.

   (b) Suppose we wish to search a linked list of length n, where each element contains a key k along with a hash value $h(k)$. Each key is a long character string. How might we take advantage of the hash values when searching the list for an element with a given key?

   (c) Given a graph $G = (V, E)$, show that if $v_1, \ldots, v_k \in V$ is the sequence of vertices of a shortest path between v_1 and v_k, then $v_1, \ldots v_{k-1}$ is the sequence of vertices of a shortest path between v_1 and v_{k-1}.

2. (a) 22.3-2

   (b) 22.4-1

3. Describe an $O(|V| + |E|)$ algorithm for the following problem: Given an undirected graph $G = (V, E)$ as adjacency lists, determine whether we can paint each vertex red or blue so that adjacent vertices get different colors. If such a coloring exists, the algorithm outputs one such coloring. (Hint: Breadth-first search).