

Analysis of Loops
Exercises (Corrected, Sept. 4)

Give the asymptotic running time of each the following functions in Θ notation. Justify your answer. (Show your work.)

1.

```

Func1(n)
1 s ← 0;
2 for i ← 3 to n2 do
3   for j ← 7 to 2i⌊log5(i)⌋ do
4     s ← s + i - j;
5   end
6 end
7 return (s);

```

2.

```

Func2(n)
1 s ← 0;
2 for i ← 3 to ⌊√n⌋ do
3   j ← i3;
4   while (j ≥ i) do
5     s ← s + i - j;
6     j ← j - 4;
7   end
8 end
9 return (s);

```

/ Note: Subtraction */*

3.

```

Func3(n)
1 s ← 0;
2 for i ← 2n to n2 do
3   for j ← i to n2 do
4     for k ← 6i to 6i + 21 do
5       s ← s + i - j + k;
6     end
7   end
8 end
9 return (s);

```

4.

```

Func4(n)
1 s ← 0;
2 for i ← ⌊n/2⌋ to ⌊4n√n⌋ do
3   for j ← 3 to i do
4     for k ← j to i do
5       s ← s + i - j + k;
6     end
7   end
8 end
9 return (s);

```

5.

```

    Func5( $n$ )
1   $s \leftarrow 0$ ;
2   $i \leftarrow 5$ ;
3  while  $i \leq n^2$  do
4       $j \leftarrow n$ ;
5      while  $j \geq 9$  do
6           $j \leftarrow j - \lceil \log_2(n) \rceil$ ;
7           $s \leftarrow s + i - j$ ;
8      end
9       $i \leftarrow i + \lceil \sqrt{n} \rceil$ ;
10 end
11 return ( $s$ );

```

6.

```

    Func6( $n$ )
1   $s \leftarrow 0$ ;
2   $i \leftarrow 5$ ;
3  while  $i \leq n^{3/2}$  do
4       $j \leftarrow 7$ ;
5      while  $j \leq i^3$  do
6           $j \leftarrow j + i$ ;
7           $s \leftarrow s + i - j$ ;
8      end
9       $i \leftarrow i + n$ ;
10 end
11 return ( $s$ );

```

7.

```

    Func7( $n$ )
1   $s \leftarrow 0$ ;
2   $i \leftarrow 1$ ;
3  while ( $i < \lfloor 6n^{3/2} \rfloor$ ) do
4      for  $j \leftarrow 1$  to  $i$  do
5           $s \leftarrow s + i - j$ ;
6      end
7       $i \leftarrow 7 * i$ ;
8  end
9  return ( $s$ );

```

/* Note: Multiplication */

8.

```

    Func8( $n$ )
1   $s \leftarrow 0$ ;
2  for  $i \leftarrow n$  to  $2n^2$  do
3       $j \leftarrow 7$ ;
4      while ( $j < 3i$ ) do
5           $s \leftarrow s + i - j$ ;
6           $j \leftarrow 3 * j$ ;
7      end
8  end
9  return ( $s$ );

```

/* Note: Multiplication */

9.

```

    Func9(n)
  1  $s \leftarrow 0$ ;
  2  $i \leftarrow n$ ;
  3 while ( $i < 5n^3$ ) do
  4    $j \leftarrow 3n^3$ ;
  5   while ( $j > 18$ ) do
  6      $s \leftarrow s + i - j$ ;
  7      $j \leftarrow \lfloor j/4 \rfloor$ ;
  8   end
  9    $i \leftarrow 4 * i$ ;
10 end
11 return ( $s$ );

```

/* Note: Division */

/* Note: Multiplication */

10.

```

    Func10(n)
  1  $s \leftarrow 0$ ;
  2  $i \leftarrow n$ ;
  3 while ( $i \leq \lfloor n \log_5(n) \rfloor$ ) do
  4    $j \leftarrow 9$ ;
  5   while ( $j < i^2$ ) do
  6      $s \leftarrow s + i - j$ ;
  7      $j \leftarrow 3 * j$ ;
  8   end
  9    $i \leftarrow i + 4$ ;
10 end
11 return ( $s$ );

```

/* Note: Multiplication */

/* Note: Addition */

11.

```

    Func11(n)
  1  $s \leftarrow 0$ ;
  2  $i \leftarrow 1$ ;
  3 while ( $i < 3n$ ) do
  4    $j \leftarrow 5$ ;
  5   while ( $j < n^2$ ) do
  6      $s \leftarrow s + i - j$ ;
  7      $j \leftarrow j + i$ ;
  8   end
  9    $i \leftarrow (2.5) * i$ ;
10 end
11 return ( $s$ );

```

/* Note: Addition */

/* Note: Multiplication */

12.

```

  1  $s \leftarrow 0$ ;
  2  $i \leftarrow 1$ ;
  3 while ( $i < n^2$ ) do
  4    $j \leftarrow 2n^3$ ;
  5   while ( $j > n$ ) do
  6      $s \leftarrow s + i - j$ ;
  7      $j \leftarrow j - 3$ ;
  8   end
  9    $i \leftarrow 4 * i$ ;
10 end
11 return ( $s$ );

```

/* Note: Subtraction */

/* Note: Multiplication */