

Probabilistic Analysis
Exercises

For each of the following problems, simplify and express your answer as $\Theta(n^k)$ or $\Theta(n^k(\log n))$ wherever possible. If the asymptotic running time is exponential, then just give exponential lower bounds.

Random(n) generates a random number between 1 and n with uniform distribution (every integer between 1 and n is equally likely.) **CoinFlip**() returns **heads** or **tails** with equal probability.

1. Consider the following function:

```
Func1(A, n)
/* A is an array of integers */
1 s ← 0;
2 k ← Random(n);
3 for i ← 1 to k do
4   | j ← 1;
5   | while (j < k) do
6   |   | s ← s + A[i] * A[j];
7   |   | j ← 2 * j;
8   |   end
9   end
10 return (s);
```

- (a) What is the asymptotic worst case running time of **Func1**?
(b) What is the asymptotic expected running time of **Func1**? Justify your solution.

2. Consider the following function:

```
Func2(A, n)
/* A is an array of integers */
1 s ← A[1];
2 k ← Random(n);
3 if (k < log2(n)) then
4   | for i ← 1 to n do
5   |   | j ← 1;
6   |   | while (j < n) do
7   |   |   | s ← s + A[i] * A[j];
8   |   |   | j ← 2 * j;
9   |   |   end
10  |   end
11 end
12 return (s);
```

- (a) What is the asymptotic worst case running time of **Func2**?
(b) What is the asymptotic expected running time of **Func2**? Justify your solution.

3. Consider the following function:

```
Func3(A, n)
/* A is an array of integers
1 if (n ≤ 10) then return (A[1]);
2 for i ← 1 to ⌊√n⌋ do
3   | A[i] ← A[i] - A[⌊i * √n⌋];
4 end
5 s ← A[1];
6 k ← Random(n);
7 if (k < 2n/3) then
8   | s ← s + Func3(A, n - 5);
9 end
10 return (s);
```

*/

- (a) What is the asymptotic worst case running time of `Func3`? Justify your solution.
- (b) What is the asymptotic expected running time of `Func3`? Justify your solution.

4. Consider the following function:

```
Func4(A, n)
/* A is an array of integers
1 if (n ≤ 10) then return (A[1]);
2 c1 ← CoinFlip();
3 c2 ← CoinFlip();
4 s ← A[k];
5 if (c1 = c2) then
6   | s ← s + Func4(A, n - 4) + Func4(A, n - 7);
7 end
8 return (s);
```

*/

- (a) What is the asymptotic worst case running time of `Func4`? Justify your solution.
- (b) What is the asymptotic expected running time of `Func4`? Justify your solution.

5. Consider the following function:

```
Func5(A, n)
/* A is an array of integers
1 if (n ≤ 10) then return (A[1]);
2 s ← 0;
3 for i ← 1 to n do
4   | A[i] ← A[i] + A[n - i + 1];
5   | s ← s + A[i];
6 end
7 for i ← 1 to 4 do
8   | c ← CoinFlip();
9   | if (c = heads) then
10    | | s ← s + Func5(A, ⌊n/4⌋);
11    | end
12 end
13 return (s);
```

*/

- (a) What is the asymptotic worst case running time of `Func5`? Justify your solution.
- (b) What is the asymptotic expected running time of `Func5`? Justify your solution.