CSE 2331 - Problem Set 5
Due beginning of lecture on October 22nd

Problem numbers are from the third edition of "Introduction to algorithms". If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, etc.) is prohibited.

1. Construct a binary search tree (BST) storing the keys 1, 6, 3, 10, 7, 2, 8, 9, 4. Given your tree, explain what happens when you `TREE-DELETE` the root and give the resulting BST.

2. Construct a red-black tree (RBT) storing the keys 1, 6, 3, 10, 7, 2, 8, 9, 4. Given your tree, explain what happens when you `RB-INSERT` key 5. You can use the convention from the lecture, where a pair of concentric circles denotes a black node and a single circle denotes a red node.

3. Describe a red-black tree on n keys that realizes the largest possible ratio of red internal nodes to black internal nodes. What is this ratio? What tree has the smallest possible ratio, and what is the ratio?

4. Given a graph $G = (V, E)$, show that if $v_1, \ldots, v_k \in V$ is the sequence of vertices of a shortest path between v_1 and v_k, then $v_1, \ldots v_{k-1}$ is the sequence of vertices of a shortest path between v_1 and v_{k-1}.

5. Describe an $O(|V| + |E|)$ algorithm for the following problem: Given an undirected graph $G = (V, E)$ as adjacency lists, determine whether we can paint each vertex red or blue so that adjacent vertices get different colors. If such a coloring exists, the algorithm outputs one such coloring. (Hint: Breadth-first search).