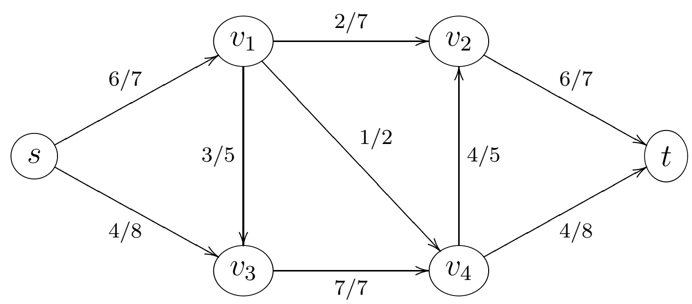
CSE 2331 - Problem Set 7
Due beginning of lecture on December 1st

Problem numbers are from the third edition of "Introduction to algorithms". If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, etc.) is prohibited.

1. Consider the following network and flow.

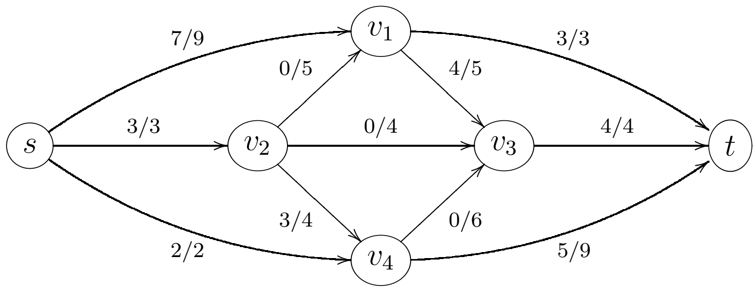


   (a) Draw the residual network for this flow.

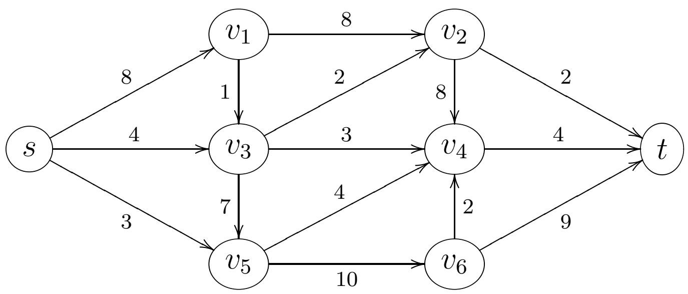
   (b) Given an augmenting path for this flow.

   (c) Increase the flow by the maximum possible value along the augmenting path and draw the resulting flow network and flow.

2. Prove that the following flow is maximum. (Hint: Use the max-flow min-cut theorem.)



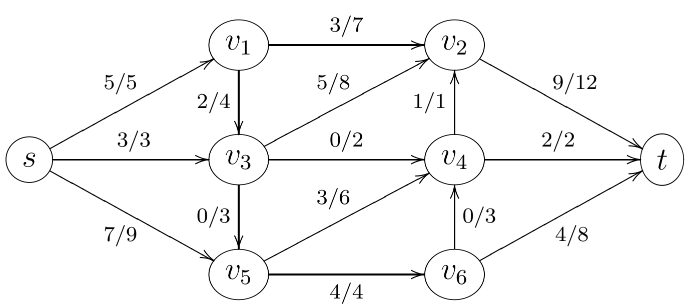
3. Consider the following flow network. (The values are edge capacities.)



(a) What is the value of the capacity of the cut (S, T) where $S = \{s, v_1, v_2, v_4\}$ and $T = \{v_3, v_5, v_6, t\}$?

(b) Prove that the cut (S, T) where $S = \{s, v_1, v_2, v_4\}$ and $T = \{v_3, v_5, v_6, t\}$ is a minimum cut? (Hint: Use the max-flow min cut theorem.)

4. Given the following maximum flow in a flow network, give the minimum cut whose value equals the maximum flow. (If you can't find the minimum cut, compute the residual network and use it to find the minimum cut.)



5. We say that a bipartite graph $G = (V, E)$, where $V = L \cup R$, is d-regular if every vertex $v \in V$ has degree exactly d. Every d-regular bipartite graph has $|L| = |R|$. Prove that every d-regular bipartite graph has a matching of cardinality $|L|$ by arguing that a minimum cut of the corresponding flow network has capacity $|L|$.