CSE 2331 - Problem Set 8
Due beginning of lecture on December 8st

Problem numbers are from the third edition of "Introduction to algorithms". If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, etc.) is prohibited.

1. Consider the following boolean expression:

   $$\phi(x_1, x_2, x_3) = (x_1 \vee x_2 \vee \overline{x_3}) \wedge (x_1 \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3}) \wedge (x_1 \vee \overline{x_2} \vee x_3).$$

   (a) Give the graph G corresponding to ϕ in the reduction of 3-SAT to CLIQUE.

   (b) Give a clique of size four in G and a corresponding truth assignment which satisfies ϕ. (A clique may correspond to more than one truth assignment.)

2. Let

   $$\text{DOUBLE-SAT} = \{\langle\phi\rangle : \phi \text{ is a boolean formula that has at least two satisfying assignments }\}.$$

   (a) Show that DOUBLE-SAT is in NP.

   (b) Show that 3-SAT reduces to DOUBLE-SAT in polynomial time.

   (c) Conclude that DOUBLE-SAT is NP-complete.

3. Assume that problem Q_1 reduces to problem Q_2 in polynomial time and both Q_1 and Q_2 are in NP.

   (a) If Q_1 can be solved in polynomial time, what can be concluded about Q_2? (The answer may be nothing.)

(b) If Q_2 can be solved in polynomial time, what can be concluded about Q_1? (The answer may be nothing.)

(c) If Q_1 is NP-complete, what can be concluded about Q_2? (The answer may be nothing.)

(d) If Q_2 is NP-complete, what can be concluded about Q_1? (The answer may be nothing.)