1. A Turing machine with stay put instead of left is similar to an ordinary Turing machine, but the transition function has the form

\[ \delta : Q \rightarrow Q \times \Gamma \times \{R, S\}. \]

At each point the machine can move its head right or let it stay in the same position. Show that this Turing machine variant is not equivalent to the usual version. What class of languages do these machines recognize?

2. Show that every infinite Turing-recognizable language has an infinite decidable subset.

3. Let \( C \) be a language. Prove that \( C \) is Turing-recognizable iff a decidable language \( D \) exists such that \( C = \{ x : \exists y(\langle x, y \rangle \in D) \} \).

4. Prove that there exists an undecidable subset of \( \{1\}^* \).