Problem numbers are from the third edition of Sipser’s book. If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited.

1. Give an implementation-level description and a formal description (i.e. including the state diagram of the transition function) of a TM that recognizes

\{u \# v : u, v \in \{0, 1\}^* \text{ and } u \text{ is } v \text{ reversed}\}

2. Show that the collection of decidable languages is closed under the operations of

   (a) complementation.

   (b) intersection.

3. Show that the collection of recognizable languages is closed under the operations of

   (a) concatenation (Given languages A, B, their concatenation is \(A \circ B := \{xy : x \in A, y \in B\}\), where \(xy\) is the concatenation of strings \(x, y\)).

   (b) intersection.

4. Prove that the union of countably many countable sets is countable.
5. Explain why the following is not a description of a legitimate Turing machine.

\[ M_{\text{bad}} = \text{“On input } \langle p \rangle, \text{ a polynomial over variables } x_1, \ldots, x_k:\]
1. Try all possible setting of \( x_1, \ldots, x_k \) to integer values.
2. Evaluate \( p \) on all of these settings.
3. If any of these settings evaluates to 0, \textit{accept}; otherwise, \textit{reject}.”