Problem numbers are from the third edition of Sipser’s book. If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited.

1. If \( A \leq_m B \) and \( B \) is a regular language, does that imply that \( A \) is a regular language? Why or why not? If yes, prove your answer, if no, show a counterexample.

2. Show that \( A_{TM} \) is not mapping reducible to the complement of \( A_{TM} \). In other words, show that no computable function reduces \( A_{TM} \) to the complement of \( A_{TM} \). (Hint: Use a proof by contradiction, and facts you already know about \( A_{TM} \) and the complement of \( A_{TM} \).)

3. Let \( \mathcal{P} = \bigcup_{k=1}^{\infty} \text{TIME}(n^k) \). Show that \( \mathcal{P} \) is closed under union, concatenation and complement.

4. We will define a nondeterministic analog of complexity class \( \text{TIME}(f(n)) \).

   Let
   \[
   \text{NTIME}(f(n)) = \{ L : L \text{ is a language decided by an} \ O(f(n)) \text{ time nondeterministic Turing machine} \}.
   \]

   Let \( \mathcal{NP} = \bigcup_{k=1}^{\infty} \text{NTIME}(n^k) \). Show that \( \mathcal{NP} \) is closed under union and concatenation.

5. Recall the definition of \( \mathcal{NP} \) from problem 4. Call graphs \( G \) and \( H \) isomorphic if the nodes of \( G \) may be reordered so that it is identical to \( H \). Let \( \text{ISO} = \{ \langle G, H \rangle : G \text{ and } H \text{ are isomorphic graphs} \} \). Show that \( \text{ISO} \in \mathcal{NP} \).