CSE 3321 - Problem Set 8
Due beginning of lecture on December 3rd

Problem numbers are from the third edition of Sipser's book. If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited.

1. If $A \leq_m B$ and B is a regular language, does that imply that A is a regular language? Why or why not? If yes, prove your answer, if no, show a counterexample.

2. Show that A_{TM} is not mapping reducible to the complement of A_{TM}. In other words, show that no computable function reduces A_{TM} to the complement of A_{TM}. (Hint: Use a proof by contradiction, and facts you already know about A_{TM} and the complement of A_{TM}.)

3. Let $\mathcal{P} = \cup_{k=1}^{\infty} \mathcal{TIME}(n^k)$. Show that $\mathcal{P}$ is closed under union, concatenation and complement.

4. We will define a nondeterministic analog of complexity class $\mathcal{TIME}(f(n))$. Let

   $$\mathcal{NTIME}(f(n)) = \{L : L \text{ is a language decided by an } O(f(n)) \text{ time nondeterministic Turing machine}\}.$$

   Let $\mathcal{NP} = \cup_{k=1}^{\infty} \mathcal{NTIME}(n^k)$. Show that $\mathcal{NP}$ is closed under union and concatenation.

5. Recall the definition of $\mathcal{NP}$ from problem 4. Call graphs G and H *isomorphic* if the nodes of G may be reordered so that it is identical to H. Let ISO $= \{\langle G, H\rangle : G \text{ and } H \text{ are isomorphic graphs}\}$. Show that ISO $\in \mathcal{NP}$.